PHY 6645 Fall 2001 – Homework 7

Due at the start of class on Thursday, November 1. No credit will be available for solutions submitted after the start of class on Tuesday, November 6.

Answer all questions. To gain full credit you should explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

1. The motion of a particle in one dimension is described by the Hamiltonian

$$H = P^2/2m + V_0 \cos \omega t.$$

   (a) Solve this problem by integrating the Schrödinger equation to find the Schrödinger-picture wave function $\psi_S(x,t)$.

   (b) Reformulate the problem in the Heisenberg picture. Find the wave function $\psi_H(x,t)$ and calculate the matrix elements $\langle p|X_H(t)|p'\rangle$ and $\langle p|P_H(t)|p'\rangle$.

   (c) Reformulate the problem in the interaction picture, choosing $H_S^{(0)} = P^2/2m$ and $H_S^{(I)} = V_0 \cos \omega t$. Find the wave function $\psi_I(x,t)$ and calculate the matrix elements $\langle p|X_I(t)|p'\rangle$ and $\langle p|P_I(t)|p'\rangle$.

2. A two-state system is described by the Hamiltonian

$$H = \hbar\omega(|1\rangle\langle 2| + |2\rangle\langle 1|),$$

where $|1\rangle$ and $|2\rangle$ are orthonormal basis vectors. Let $|\pm\hbar\omega\rangle$ be orthonormal eigenkets of H, which satisfy $H\,|\pm\hbar\omega\rangle = \pm\hbar\omega\,|\pm\hbar\omega\rangle$. Consider also an operator

$$\Omega = \omega_1|1\rangle\langle 1| + \omega_2|2\rangle\langle 2|.$$

For each of the cases (a)–(d) below, calculate $\langle\Omega(t)\rangle$, the expectation value of Ω at time t, using the density-operator formulation of quantum mechanics.

In cases (a) and (b), you should also find $\langle\Omega(t)\rangle$ using the state-vector formulation of QM; do the calculation both in the Schrödinger picture and in the Heisenberg picture. You should get the same result for $\langle\Omega(t)\rangle$ via all three methods.

   (a) $\rho(t{=}0) = |-\hbar\omega\rangle\langle -\hbar\omega|$.

   (b) $\rho(t{=}0) = |2\rangle\langle 2|$.

   (c) $\rho(t{=}0) = a|1\rangle\langle 1| + b|2\rangle\langle 2|$.

   (d) $\rho(t{=}0) = a|1\rangle\langle 1| + b|2\rangle\langle 2| + c|-\hbar\omega\rangle\langle -\hbar\omega|$.

3. Based on Sakurai Problem 3.10.

Prove that a pure ensemble at time $t = 0$ cannot evolve into a mixed ensemble at time $t > 0$, provided that the time evolution is governed by the Schrödinger equation.