PHY 6645 Fall 2003 – Homework 3

Due by 5 p.m. on Friday, September 19. No credit will be available for homework submitted after 5 p.m. on Monday, September 22.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. Consider three possible quantizations of the classical dynamical variable $\omega = x^3p^2$: $\Omega_1 = X^3P^2$, $\Omega_2 = P^2X^3$, and $\Omega_3 = XPXPX$. Use the commutation relation between X and P to express each of the differences

   (a) $\Omega_1 - \Omega_2$,

   (b) $\Omega_1 - \Omega_3$,

   (c) $\Omega_2 - \Omega_3$,

   as a sum of terms, where each term is a product of constants and powers of X and P, with any X operators appearing to the left of any P operators. (X^2P meets the last requirement, but PX^2 does not.)

2. This problem is based on, but goes beyond, Shankar Exercise 4.2.1. Before tackling this problem, you may wish to work through Shankar's exercise and check that you recover the answers in the back of the text.

   Let L_x, L_y, and L_z be as defined in Shankar Exercise 4.2.1.

   (a) What are the possible values one can obtain if L_x is measured?

   (b) What are the possible values one can obtain if L_y is measured?

   (c) What are the possible values one can obtain if L_z is measured?

   (d) Find the normalized eigenstates of L_x in the original basis (i.e., the basis used to represent L_x, L_y, and L_z in Shankar Exercise 4.2.1). In this and subsequent parts, label each eigenstate with its eigenvalue, e.g., "$L_x = 5$".

   (e) Find the normalized eigenstates of L_y in the original basis.

   (f) Find the normalized eigenstates of L_z in the original basis.

   (g) Find the normalized eigenstates of L_y in the basis of eigenstates of L_x.

   (h) Find the representation of L_y in the basis of eigenstates of L_x.

   (i) The particle is in the state in which $L_x = 1$. What are $\langle L_y\rangle$, $\langle L_y^2\rangle$, and ΔL_y? Calculate $\langle L_y\rangle$ and $\langle L_y^2\rangle$ in two ways: (i) working in the original basis; (ii) working in the basis of eigenstates of L_x.

   (j) The particle is in the state in which $L_x = 1$. If L_y is measured, what are the possible outcomes and their probabilities?

   (k) The particle is in the state with $L_x = 1$. A measurement of L_y^2 yields the result $L_y^2 = 1$. What is the state of the system after the measurement? Give your answer in the original basis.

   (l) Following the measurement of $L_y^2 = 1$ described in part (k), L_x is measured. What are the possible outcomes and their probabilities?