PHY 6645 Fall 2003 – Homework 9

Due by 5 p.m. on Wednesday, December 3. No credit will be available for homework submitted after 5 p.m. on Friday, December 5.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. Shankar Exercise 12.3.3.
2. Shankar Exercise 12.3.7, parts (1)–(8). You may ignore the last sentence of part (8).
3. Shankar Exercise 12.5.6.
4. Merzbacher Problem 11.1: For the state represented by the wave function

$$\psi(x, y, z) = Ne^{-\alpha r^2}(x + y)z,$$

   (a) determine the normalization constant N as a function of the parameter α;

   (b) calculate the expectation values of $\mathbf{L}$ and L^2;

   (c) calculate the variances of these quantities.

5. Consider a particle of mass m in a spherical box of radius a, i.e., a particle in a potential

$$V(\mathbf{r}) = \begin{cases} 0 & |\mathbf{r}| < a, \\ \infty & |\mathbf{r}| \geq a. \end{cases}$$

   Find the ten lowest-energy eigenstates of this problem. For each eigenstate, list the quantum numbers, the energy in units of $\hbar^2/ma^2$, and the wave function in terms of spherical Bessel functions and spherical harmonics. (Do not bother to normalize the wave functions.)

   Hint: You will need to look up the positions x of the first few zeros of $j_l(x)$. These can be found in standard references, e.g., M. Abramowitz and I. E. Stegun, *Handbook of Mathematical Functions* (Dover Publications, New York, 1974).

6. Consider a particle in the hydrogenic state (n, l), which has energy E_n. Let r_n be the radius about the proton of the surface of classical turning points for an electron having energy E_n. Let $P_{n,l}(R)$ be the probability that the quantum-mechanical electron will be found at a distance from the proton greater than R.

   Calculate $P_{n,l}(a_0)$ and $P_{n,l}(2r_n)$ for the $n = 1$ and $n = 2$ levels. In each case, your final answer should be a pure number.