# PHY 6645 Fall 2003 – Homework 10

**Due by 5 p.m. on Wednesday, December 10.** The 25% penalty usually levied for late homework will not be applied on this assignment. However, no credit will be available for homework submitted after 5 p.m. on Friday, December 12.

*Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.*

1. Shankar Exercise 14.3.7. (Since the answers are in the book, you must show all your working to receive any credit.)

2. Based on Sakurai Problems 3.9 and 3.11:

   (a) Consider a pure ensemble of spin-$\frac{1}{2}$ systems. Suppose the expectation values $\langle S_x \rangle$ and $\langle S_z \rangle$ and the sign of $\langle S_y \rangle$ are known. Show how we may determine the density matrix for this state. Why is it not necessary to know the magnitude of $\langle S_y \rangle$?

   (b) Consider a mixed ensemble of spin-$\frac{1}{2}$ systems. Suppose that $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$ are known. Show how we may determine the density matrix for this state.

   (c) Consider an ensemble of spin-1 systems. How many independent (real) parameters are needed to characterize the density matrix? What must we know in addition to $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$ to characterize the state completely?

3. Ballentine Problem 7.9: Consider a system of two spin-$\frac{1}{2}$ particles. Calculate the eigenvalues and eigenvectors of the operator $\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)}$. Use the product vectors $|m_1\rangle \otimes |m_2\rangle$ as basis vectors.

4. As was shown in class, the sum $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$ of two independent (commuting) spins $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ also behaves like a spin, i.e., its components obey angular momentum commutation relations.

   (a) Show that the difference $\boldsymbol{D} = \boldsymbol{S}_1 - \boldsymbol{S}_2$ does not behave like a spin.

   Now consider a system consisting of two spin-$\frac{1}{2}$ degrees of freedom.

   (b) Find an orthonormal eigenbasis of $D_z$ for this system. Express your eigenstates in the basis of product states $|m_1\rangle \otimes |m_2\rangle$.

   (c) Find an orthonormal eigenbasis of $D^2 = \boldsymbol{D} \cdot \boldsymbol{D}$ for this system.

   (d) Show that it is not possible to construct a simultaneous eigenbasis of $D_z$ and $D^2$ for this system.

5. Based on Sakurai Problem 3.20: We are to add angular momenta $j_1 = 1$ and $j_2 = 1$ to form $j = 2$, 1, and 0 states. Express each of the nine $\{j, m\}$ eigenkets as a sum of product basis states $|j_1, m_1; j_2, m_2\rangle$. You should derive the relevant Clebsch-Gordan coefficients, **not** merely quote them.

   You are, of course, free to check the correctness of your answers against precalculated CG coefficients. A handy one-page table of CG coefficients is available in Postscript format at `http://pdg.lbl.gov/2002/clebrpp.ps` and in PDF format at `http://pdg.lbl.gov/2002/clebrpp.pdf`. An online CG calculator is available at `http://www.ph.surrey.ac.uk/~phs3ps/cleb.html`, which also has links to several other calculators.