PHY 6645 Fall 2003 K. Ingersent

The Postulates of Nonrelativistic Quantum Mechanics

- The following postulates are based on those appearing in Shankar Section 4.1. They apply to *pure states* of a single, pointlike particle in one spatial dimension. Generalizations to *mixed states*, multiple particles and/or higher dimensions will be discussed later.

I. The *state* of the particle is represented by a vector $|\psi(t)\rangle$ in a *physical Hilbert space*.

II. To each *dynamical variable* ω there corresponds a linear Hermitian operator Ω acting in the physical Hilbert space.

III. If the particle is in a state $|\psi\rangle$, then *measurement* of the dynamical variable ω will yield one of the eigenvalues ω_m of Ω. After the measurement, the state of the particle will have been *reduced* from $|\psi\rangle$ to an eigenvector $|\omega_m\rangle$ corresponding to the eigenvalue ω_m. The *probability* for this outcome is proportional to $|\langle\omega_m|\psi\rangle|^2$.

IV. The state vector obeys the Schrödinger equation of motion:

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle,$$

where H is the *Hamiltonian operator* and $\hbar = h/2\pi$, h being Planck's constant.

- In *simple cases*, the quantum mechanical description of a particle can be derived from the classical Hamiltonian formulation using the following quantization procedure:

1. Replace the position x by the operator X having the eigenkets $|x\rangle$ introduced in Ch. 1, such that $\langle x|X|x'\rangle = x\delta(x-x')$.
2. Replace the momentum p by the operator $P = \hbar K$, where K was introduced in Ch. 1. Note that $\langle x|P|x'\rangle = -i\hbar\delta'(x-x') = -i\hbar\delta(x-x')\partial/\partial x'$.
3. Construct all other operators (including the Hamiltonian H) from the corresponding classical dynamical variables via the recipe

$$\Omega(X, P, t) = \omega(x \to X, p \to P, t).$$

- The postulates and quantization prescription given above are not very useful without a definition of each of the italicized terms. The *physical Hilbert space* has already been defined, while *dynamical variables* should be familiar from classical mechanics. The meaning of the remaining terms will be discussed in class. We will see that Postulate III is not universally accepted.