PHY 6646 Spring 2002 – Mid-Term Exam 1

Instructions: Attempt both Question 1 (worth 20 points) and Question 2 (worth 30 points). The maximum score for each part of each question is shown in square brackets. To gain full credit you should explain your reasoning and show all working. Please write neatly and remember to include your name on the front page of your answers.

You are allowed to consult Shankar's *Principles of Quantum Mechanics*, your lecture notes, and homework solutions from this course. You may not consult any other books or papers, or communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

Questions 1 and 2 both concern a particle of mass m moving in the one-dimensional potential

$$V(x) = \begin{cases} \kappa(x+a) & \text{for } -a < x < 0, \\ \kappa(x-a) & \text{for } 0 < x < a, \\ +\infty & \text{for } |x| > a, \end{cases}$$

where κ is real and positive.

1. You should be able to answer this question with very little mathematical working.
   (a) Treat the terms in V proportional to κ as a perturbation about the standard particle-in-a-box problem (which corresponds to $\kappa = 0$). Use first-order perturbation theory to approximate the bound-state energies E_n, $n = 0, 1, 2, \ldots$ [8]
   (b) Give a rigorous upper bound on the ground-state energy, E_0. [6]
   (c) Provide a criterion for the range of κ within which perturbation theory should be valid. Your criterion may be quite rough, but it should not include any unevaluated sums or integrals. [6]

2. This question requires more extensive algebra. Attempt to evaluate all integrals in closed form.
   (a) Apply the WKB approximation to the potential above to derive an algebraic equation satisfied by the bound-state energies E_n in the limit that $E_n \gg V(x)$ for all $|x| < a$. [10]
   Hints: (i) Be sure to take into account the sharp rise in the potential at $x = \pm a$. (ii) The equation you should obtain cannot be solved for E_n in closed form.
   (b) Expand the equation obtained in part (a) for the case that $\kappa a \ll E_n$. Hence evaluate the WKB expression for E_n up to (and including) the leading κ dependence. Is your result consistent with that of Question 1(a)? [10]
   (c) Assume instead that κ entering the potential above is very large. Obtain a reasonable estimate of the ground-state energy E_0 using an appropriate method of your choosing. Here, a reasonable estimate is one that is likely to lie within 20% of the true value. [10]