

PHY 6646 Spring 2002 – Homework 7

Due by 5 p.m. on Friday, March 22. No credit will be available for solutions submitted after 5 p.m. on Monday, March 25.

Answer all questions. To gain full credit you should explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

1. Shankar Exercise 17.2.4.

Generalize this problem to three dimensions, proving the Thomas-Reiche-Kuhn sum rule in the form

$$\sum_m f_{mn}^{(j)} = 1, \quad (1)$$

where the *oscillator strength* $f_{mn}^{(j)}$ ($j = 1, 2, \text{ or } 3$) is defined as

$$f_{mn}^{(j)} = \frac{2m}{\hbar^2} (E_m - E_n) |\langle m | r_j | n \rangle|^2.$$

Background note: For an atom or ion having N_e electrons, Eq. (1) generalizes to

$$\sum_m f_{mn}^{(j)} = N_e, \quad (2)$$

where the states $|m\rangle$ and $|n\rangle$ are many-electron states of the total Hamiltonian. Equation (2) is called the “ f -sum rule,” although this name is often applied to a slightly different formulation of the equation.

2. Merzbacher Exercise 19.5: For a hydrogen atom that is initially in the $1S$ ground state, compute the z -axis oscillator strengths $f_{mn}^{(3)}$ for transitions to the lowest three excited states. What fraction of the total do these oscillator strengths add up to?
3. Based on Merzbacher Exercise 19.7: For the $1S \rightarrow 2P$ transitions in the hydrogen atom, evaluate the total integrated absorption cross section, assuming that the incident radiation is all traveling along the z axis.
4. Based on Sakurai Problem 5.35: The ground state of a hydrogen atom is subjected to a time-dependent potential

$$V(\mathbf{r}, t) = V_0 \cos(kz - \omega t).$$

Using time-dependent perturbation theory, obtain an expression for the transition rate at which an electron is emitted in a plane-wave state of momentum \mathbf{p} . Do not use the electric dipole approximation. Express your answer in terms of the spherical polar components of the momentum: $\mathbf{p} = (p, \theta, \phi)$.