PHY 6646 Spring 2002 – Homework 9

Due by 5 p.m. on Friday, April 19. No credit will be available for solutions submitted after 5 p.m. on Monday, April 22.

Answer all questions. To gain full credit you should explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

1. Shankar Exercise 10.3.4.

2. Suppose that a system of identical fermions has available just three single-particle states, denoted $|\alpha\rangle$, $\alpha = 1,\ 2,\ 3$. Simplify each of the expressions below so that, wherever possible, it assumes the form of a many-particle basis state $|n_1, n_2, n_3\rangle$.

   (a) $c_3|1,0,1\rangle$

   (b) $c_1^\dagger|1,0,1\rangle$

   (c) $c_2 c_1^\dagger c_2^\dagger|0\rangle$

3. Standardize the following products of bosonic (a) or fermionic (c) creation and annihilation operators. By "standardize" is meant (i) reducing the number of operators in each product to the minimum possible, e.g., by eliminating terms such as $c_\alpha c_\alpha$; (ii) placing all creation operators to the **left** of annihilation operators; (iii) among the creation operators, placing those for low-index single particle states to the **left** of those for high-index states; and (iv) among the annihilation operators, placing those for low-index single particle states to the **right** of those for high-index states.

   (a) $a_1 a_2^\dagger a_1$

   (b) $c_1 c_2^\dagger c_1$

   (c) $c_3 c_1^\dagger c_2 c_1 c_1^\dagger c_2^\dagger c_1$

   (d) $a_2 a_2^\dagger a_2^\dagger a_2 a_2^\dagger$

4. Based on Merzbacher Exercise 21.8: Prove from the (anti)commutation relations that for bosons
$$\langle 0|a_i a_j a_k^\dagger a_l^\dagger|0\rangle = \delta_{j,k}\delta_{i,l} + \delta_{i,k}\delta_{j,l},$$
   while for fermions
$$\langle 0|c_i c_j c_k^\dagger c_l^\dagger|0\rangle = \delta_{j,k}\delta_{i,l} - \delta_{i,k}\delta_{j,l}.$$

5. Show that the field operators obey
$$\begin{aligned}
[\psi_\sigma(\mathbf{r}), \psi_{\sigma'}(\mathbf{r}')]_\pm &= 0, \\
[\psi_\sigma^\dagger(\mathbf{r}), \psi_{\sigma'}^\dagger(\mathbf{r}')]_\pm &= 0, \\
[\psi_\sigma(\mathbf{r}), \psi_{\sigma'}^\dagger(\mathbf{r}')]_\pm &= \delta(\mathbf{r}-\mathbf{r}')\delta_{\sigma,\sigma'},
\end{aligned}$$
   where $[A,B]_-$ is the commutator of A and B, and $[A,B]_+$ is the anticommutator of A and B.