PHY 6646 Spring 2003 – Mid-Term Exam 2

Instructions: Attempt both Question 1, which is worth 45 points, and Question 2, which is worth 55 points. The maximum score for each part of each question is shown in square brackets. To gain full credit you should explain your reasoning and show all working. Please write neatly and remember to include your name on the front page of your answers.

Please read carefully: During this exam, you may use (and quote results from) (i) Shankar's *Principles of Quantum Mechanics* (ii) lecture notes from this course, and (iii) homework solutions from this course. You are not permitted to consult any other books, notes, or papers, or to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

1. A particle of mass m is confined to a two-dimensional square well by the potential

$$V(x,y) = \begin{cases} \dfrac{V_1 xy}{a^2} & \text{for } |x| < a \text{ and } |y| < a, \\ \infty & \text{otherwise.} \end{cases}$$

   (a) Start by setting $V_1 = 0$. Write down the energies and wave functions of the three stationary states of lowest energy. (Take care to get this part right, because the answers are used in the remainder of the problem!) [10]

   (b) Now let V_1 be nonzero. Use perturbation theory to find the three lowest energy eigenvalues, working to first order in V_1. Give the corresponding eigenfunctions to zeroth order in V_1. [30]

   (c) Without performing any calculations, what can you say about the second-order correction to the ground-state energy? [5]

   You may find the following useful:

$$\begin{aligned} \int \theta \, \sin\theta \, d\theta &= \sin\theta - \theta\cos\theta, \\ 8\int \theta \, \sin^2\theta \, d\theta &= 2\theta^2 - 2\theta\sin 2\theta - \cos 2\theta, \\ 36\int \theta \, \sin^3\theta \, d\theta &= 27\sin\theta - \sin 3\theta - 27\theta\cos\theta + 3\theta\cos 3\theta. \end{aligned}$$

Turn over the page for Question 2.

2. A one-dimensional system containing a particle of mass m is initially described by the Hamiltonian $H_0 = P^2/2m + \frac{1}{2}m\omega_0^2 X^2$. Here, X and P are the position and momentum operators, respectively, and $\omega_0 > 0$ is an angular frequency. Just before time $t = 0$, this system is in its ground state.

   For each perturbation $H_1(t)$ described below, let $P_n(t, t')$ be the probability at time t that the system is in the n'th excited state of $H(t') = H_0 + H_1(t')$, with $n = 0$ representing the ground state. (Note carefully the distinction between t and t'.)

   (a) Suppose that the system is subjected to the perturbing Hamiltonian $H_1(t) = \lambda X^4 \theta(t) \tanh(t/\tau)$, where $\theta(t)$ is the step (Heavyside) function, $\lambda > 0$, and $\omega_0 \tau \sim 10^6$. Calculate approximate values for $P_0(\infty, \infty)$ and $P_1(\infty, \infty)$. [10]

   (b) Suppose instead that the system is subjected to the perturbing Hamiltonian $H_1(t) = -\frac{3}{8}m\omega_0^2 X^2 \theta(t) \tanh(t/\tau)$, where $\omega_0 \tau \sim 10^{-6}$. Calculate approximate values for $P_0(\infty, \infty)$ and $P_1(\infty, \infty)$. [25]

   (c) Finally, suppose that the system is subjected to the perturbing Hamiltonian $H_1(t) = fX\theta(t)\sin(\omega t)$, where $f > 0$ and $\omega > 0$. Working to first order in time-dependent perturbation theory, find $P_0(t, 0)$ and $P_1(t, 0)$ for small times t such that $0 < (\omega_0 + \omega)t \ll 1$. [20]