PHY 6646 Spring 2003 – Homework 1

Due by 5 p.m. on Friday, January 17. Partial credit will be available for solutions submitted by 5 p.m. on Tuesday, January 21.

Answer all questions. To gain maximum credit you should explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

1. Shankar Exercise 15.2.4.

2. Shankar Exercise 15.2.6.

   Your copy of Shankar may contain a typo in this question: "the project operators" should be "the projection operators".

3. Modified from Sakurai Problem 3.20: We are to add angular momenta $j_1 = \frac{1}{2}$ and $j_2 = \frac{3}{2}$ to form $j = 1$ and 2 states. Express each of the eight $\{j, m\}$ eigenkets as a sum of product basis states $|j_1, m_1; j_2, m_2\rangle$. You should derive the relevant Clebsch-Gordan coefficients, **not** merely quote them.

   Comment: You are, of course, free to check the correctness of your answers against precalculated CG coefficients. A handy one-page table of CG coefficients is available in Postscript format at `http://pdg.lbl.gov/2002/clebrpp.ps` and in PDF format at `http://pdg.lbl.gov/2002/clebrpp.pdf`. An online CG calculator is available at `http://www.ph.surrey.ac.uk/~phs3ps/cleb.html`, which also has links to several other calculators.

4. Ballentine Problem 7.16: Consider a system of three particles of spin $\frac{1}{2}$. A basis for the the states of this system is provide by the eight product vectors $|m_1\rangle \otimes |m_2\rangle \otimes |m_3\rangle$, where the m's take on the values $\pm\frac{1}{2}$. Find the linear combinations of these product vectors that are eigenvectors of the total angular momentum operators, $\mathbf{J} \cdot \mathbf{J}$ and J_z, where $\mathbf{J} = \mathbf{S}^{(1)} + \mathbf{S}^{(2)} + \mathbf{S}^{(3)}$. Give the eigenvalues of $\mathbf{J} \cdot \mathbf{J}$ and J_z in each state.

   Hint: If you want, you can combine spins number 1 and 2, say, to form a basis of eigenkets of $|\mathbf{S}^{(1)} + \mathbf{S}^{(2)}|^2$, then combine these states with those of spin number 3. However, this problem is sufficiently simple that you should be able to construct eigenkets of $\mathbf{J} \cdot \mathbf{J}$ directly.

5. Shankar Exercise 15.3.3.

6. Shankar Exercise 15.3.4.