# PHY 6646 Spring 2003 – Homework 2

Due by 5 p.m. on Friday, January 24. Partial credit will be available for solutions submitted by 5 p.m. on Monday, January 27.

Answer all questions. To gain maximum credit you should explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

1. Adapted from Ballentine Problem 11.2: Show that the gauge-independent probability current density, $\boldsymbol{J} = \text{Re}\,(\psi^* \boldsymbol{V} \psi)$, satisfies the continuity equation $\partial|\psi|^2/\partial t + \boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$.

2. Last year, a group working in Grenoble reported [V. Nesvizhevsky et al., Nature **415**, 297 (2002), summarized in Physics Today, March 2002, pp. 20–22] the observation of low-lying stationary states of ultra-cold neutrons trapped by the earth's gravitational field above a horizontal reflecting surface. Semiclassically, the neutrons can be thought of as bouncing off this surface as shown below:
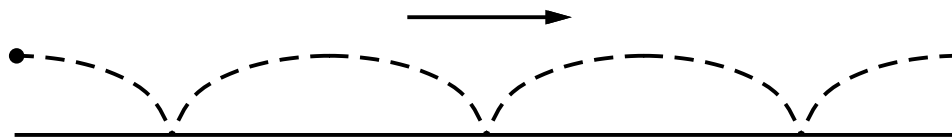
   We will analyze this problem by considering a neutron moving in a potential $V(x, y, z) = +\infty$ for $z < 0$, $V(x, y, z) = mgz$ for $z \geq 0$; here, z measures the height above the surface of the mirror.

   (a) Show that the stationary states for this problem can be written in the form $\psi(x, y, z) = \psi_x(x)\psi_y(y)\psi_z(z)$.

   (b) Show that the energy eigenvalues for this problem can be written in the form $\varepsilon(k_x, k_y, n) = \hbar^2(k_x^2 + k_y^2)/2m + \varepsilon_n$, where k_x and k_y are real numbers and n is a non-negative integer.

   (c) Find ε_0, ε_1, and ε_2. Express each answer in terms of variables introduced above and also in electron volts.

   (d) In the experiments, the neutrons have a horizontal speed of approximately 10 m/s. Estimate the total ground-state energy (of one neutron), in electron volts.

   (e) Find the height above the mirror of the classical turning point in the states $n = 0$, 1, and 2. Express each answer in terms of variables introduced above and also in meters.

   (f) Sketch the density of states, $g(\varepsilon)$ versus ε.

   Airy functions are tabulated in many standard sources, and can be calculated using packages such as Matlab and Mathematica.

3. Consider a particle of mass m and charge q moving in a region containing both a uniform magnetic field $\boldsymbol{B} = B\hat{\boldsymbol{z}}$ and a uniform electric field $\boldsymbol{E}$.

   (a) Find the energy spectrum and wave functions for the case $\boldsymbol{E} = E\hat{\boldsymbol{z}}$.

   (b) Find the energy spectrum and wave functions for the case $\boldsymbol{E} = E_x\hat{\boldsymbol{x}} + E_z\hat{\boldsymbol{z}}$.

   Hint: With the correct choice of gauge, the solutions for (b) can be deduced quite easily from those for (a).