

PHY 6646 Spring 2003 – Homework 7

Due by 5 p.m. on Monday, March 17. No credit will be available for solutions submitted after this deadline. (NB: There are no classes the week of March 10–14.)

Answer all questions. To gain full credit you should explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

1. The weak-field Zeeman effect in hydrogen: Consider the application of a weak magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ to a hydrogen atom, so that there is an additional Hamiltonian term (the Zeeman term):

$$H_Z = -\gamma(\mathbf{L} + g\mathbf{S}) \cdot \mathbf{B} = -\gamma B[J_z + (g - 1)S_z],$$

where $\gamma = -e/2mc$ (e being assumed to be positive).

- (a) For a given value of the principal quantum number n , within what range of B would you expect it to be valid to treat H_Z as a perturbation of the fine structure given in Shankar Eq. (17.3.22)?
- (b) In order to perturb about the fine structure energies, it is necessary to calculate matrix elements of S_z between states $|n, j, m_j, l, s\rangle$. [Of course, $s = \frac{1}{2}$ always in the hydrogen atom. Nonetheless, keep s as an arbitrary (integer or half-integer) quantity in this part of the question.] The Wigner-Eckart theorem comes in useful here. Use the result of Homework 1, Question 6 to show that the first-order energy shift can be written

$$\langle n, j, m_j, l, s | H_Z | n, j, m_j, l, s \rangle = g_{\text{eff}} \mu_B B m_j,$$

where $\mu_B = e\hbar/2mc$ is the Bohr magneton. Provide an explicit expression for the “Landé g -factor” g_{eff} as a function of g , j , l , and s .

- (c) Calculate the total (Coulomb + fine structure + Zeeman) ground state energy of hydrogen as a function of the field strength B .
2. Shankar Exercise 18.2.2.
 3. Sakurai Problem 5.23: A one-dimensional harmonic oscillator is in its ground state for $t < 0$. For $t \geq 0$, it is subjected to a time-dependent but spatially uniform force (not potential!) in the x -direction, $F(t) = F_0 e^{-t/\tau}$.
 - (a) Using time-dependent perturbation theory to first order, obtain the probability of finding the oscillator in its first excited state for $t > 0$. Show that the $t \rightarrow \infty$ (with τ finite) limit of your expression is independent of time. Is this reasonable or surprising?
 - (b) Can we find higher excited states?

4. Shankar Exercise 17.2.4.

Modification: In part (1), prove the Thomas-Reiche-Kuhn sum rule for an arbitrary potential $V(\mathbf{r})$ in three dimensions (rather than one dimension, as Shankar asks), namely,

$$\sum_m f_{mn}^{(j)} = 1,$$

where the *oscillator strength* $f_{mn}^{(j)}$ ($j = 1, 2, \text{ or } 3$, corresponding to $x, y, \text{ or } z$, respectively) is defined as

$$f_{mn}^{(j)} = \frac{2m}{\hbar^2} (E_m - E_n) |\langle m | r_j | n \rangle|^2.$$

Note that the oscillator strength is important for calculating the absorption cross section for electromagnetic radiation.

5. A hydrogen atom initially in its ground state is subjected to an oscillatory electric field directed along the z axis.

- (a) Identify the two states of lowest energy to which first-order transitions can take place.
- (b) Calculate the oscillator strength for each of the transitions identified in (a). What fraction of the total do these oscillator strengths add up to?