

PHY 6646 Spring 2003 – Homework 10

Due by 5 p.m. on Friday, April 18. No credit will be available for solutions submitted after 5 p.m. on Monday, April 21.

Answer all questions. To gain full credit you should explain your reasoning and show all working. Please write neatly and include your name on the front page of your answers.

1. Shankar Exercise 19.3.1.
2. Shankar Exercise 19.3.3.
3. Shankar Exercise 19.3.4.
4. Shankar Exercise 19.5.4.
5. Based on Sakurai Problem 7.9: Consider scattering from a repulsive δ -shell potential:

$$\left(\frac{2m}{\hbar^2}\right)V(r) = \gamma\delta(r - R), \quad (\gamma > 0).$$

- (a) Set up an equation that determines the s -wave phase shift δ_0 as a function of k ($E = \hbar^2 k^2 / 2m$).
- (b) Assume now that γ is very large,

$$\gamma \gg \frac{1}{R}, k.$$

Show that if $\tan kR$ is *not* close to zero, the s -wave phase shift resembles the hard-sphere result discussed in class.

- (c) Show, also for very large γ , that for $\tan kR$ close to (but not exactly equal to) zero, resonance behavior is possible; that is, $\cot \delta_0$ goes through zero from the positive side as k increases. Determine approximately the positions of the resonances, keeping terms of order $1/\gamma$. Compare these resonance positions with the bound-state energies for a particle confined *inside* a spherical box of the same radius,

$$V = \begin{cases} 0 & \text{for } r < R, \\ \infty & \text{for } r \geq R. \end{cases}$$

Also obtain an approximate expression for the resonance width Γ defined by

$$\Gamma = \frac{-2}{[d(\cot \delta_0)/dE]|_{E=E_r}}.$$