PHY 6646 Spring 2004 – Mid-Term Exam 1

Instructions: Attempt all three questions. The maximum score for each part of each question is shown in square brackets, as is the total for the question. To gain full credit you should explain your reasoning and show all working. Please write neatly and remember to include your name on the front page of your answers.

Please read carefully: During this exam, you may use Shankar's *Principles of Quantum Mechanics* and lecture notes from this course. You may also use standard mathematical tables. You may quote without proof any results given in these sources; however, you should cite the source for the result (e.g., "Shankar page 25"). You are not permitted to consult any other books, notes, or papers, or to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

1. [35 points in total] A spinless particle of mass m and charge $-e$ moves in a region containing uniform electric and magnetic fields, $\boldsymbol{E} = E\hat{\boldsymbol{z}}$ and $\boldsymbol{B} = B\hat{\boldsymbol{z}}$. Here, e, E, and B are all positive quantities.

   (a) [15 points] Assuming that the particle experiences no forces other than those produced by the electromagnetic fields described above, write down the Hamiltonian and the form of the stationary-state wave functions $\psi(x, y, z)$. Make sure that you specify which gauge you are working in, list each of the quantum numbers used to characterize the stationary states, and express the energy in terms of those quantum numbers. Do not bother about the normalization of your wave functions.

   Hint: You do not need to solve the time-independent Schrödinger equation from scratch. It is enough to combine standard results, with a brief explanation of why you are doing so.

   (b) [10 points] Suppose that the particle is confined to the region $z > 0$ by a rigid wall that occupies the entire plane $z = 0$. Assuming that the electric and magnetic fields remain the same, explain qualitatively how your answers to part (a) must be modified to deal with this new situation.

   (c) [10 points] Finally, suppose that more walls are added so that the particle is confined to the region $0 < x < L_x$, $0 < y < L_y$, $0 < z < L_z$, where L_x, L_y and L_z are all much greater than any length scales associated with the electric and magnetic fields. By imposing periodic boundary conditions at some of the walls, estimate the degeneracy of the ground state. Ignore any accidental degeneracy that arises for special values of E/B.

2. [25 points in total] Consider a particle of mass m moving in the one-dimensional potential $V(x) = -V_0 a\delta(x)$, where V_0 and a are positive, real quantities.

   (a) [20 points] Use a triangular trial wave function,

   $$\psi_\alpha(x) = \begin{cases} A(\alpha - |x|) & \text{for } |x| < \alpha, \\ 0 & \text{otherwise,} \end{cases}$$

   to obtain a variational upper bound on the ground-state energy of this system.

   (b) [5 points] What is the best possible (i.e., lowest) variational upper bound on the ground-state energy for this potential that can be obtained using any trial wave function $\psi_\gamma(x)$ that satisfies $\psi_\gamma(-x) = -\psi_\gamma(x)$?

3. [40 points in total] Consider a particle of mass m trapped by the one-dimensional potential

   $$V(x) = \begin{cases} V_0(|x/a| - 1) & \text{for } |x| < a \\ \infty & \text{otherwise} \end{cases}$$

   (a) [20 points] Show that the WKB method predicts that any negative-energy stationary states of this problem have energies

   $$E_n = (2n+1)^{2/3} \left(\frac{9\pi^2\hbar^2 V_0^2}{128 m a^2} \right)^{1/3} - V_0,$$

   where n is a non-negative integer.

   (b) [8 points] Use the WKB method to write down a condition satisfied by the positive-energy eigenvalues of this problem, i.e., $E_n > 0$.

   (c) [12 points] Obtain a closed-form expression for the energy eigenvalues E_n in the limit $E_n \gg V_0$. Compare your result with the (exact) energy eigenvalues for the case $V_0 = 0$.