

PHY 6646 Spring 2004 – Mid-Term Exam 2

Instructions: Attempt all three questions, which carry roughly equal weight. The maximum score for each part of each question is shown in square brackets. To gain full credit you should explain your reasoning and show all working. Please write neatly and remember to include your name on the front page of your answers.

Please read carefully: During this exam, you may use Shankar's *Principles of Quantum Mechanics* and lecture notes from this course. You may also use standard mathematical tables. You may quote without proof any results given in these sources; however, you should cite the source for the result (e.g., "Shankar page 25"). You are not permitted to consult any other books, notes, or papers, or to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

1. The "rigid rotator" is a simple model of a diatomic model in which two point masses are separated by a massless, rigid rod. The Hamiltonian for a rigid rotator in a constant, spatially uniform magnetic field \mathbf{B} is (neglecting both the center-of-mass motion and a term quadratic in the field)

$$H = \frac{|\mathbf{L}|^2}{2I} - \gamma \mathbf{L} \cdot \mathbf{B}, \quad (1)$$

where \mathbf{L} is the molecule's angular momentum, I is its moment of inertia, and γ is its gyromagnetic ratio. The stationary states of the rigid rotator described by Eq. (1) are simultaneous eigenkets of $|\mathbf{L}|^2$ and $\mathbf{B} \cdot \mathbf{L}$, having energies

$$E_{l,m} = \frac{\hbar^2 l(l+1)}{2I} - \gamma |\mathbf{B}| \hbar m, \quad (2)$$

where l and m are both integers.

- (a) [25 points] Suppose that \mathbf{B} has Cartesian coordinates $(B_1, 0, B_0)$, where $|B_1| \ll |B_0|$. Calculate the energy eigenvalues of the Hamiltonian (1) to second order in perturbation theory, taking $B_1 = 0$ as your unperturbed starting point.
- (b) [8 points] Show that the energies you calculated in (a) coincide with those obtained by expanding the exact eigenvalues [given by Eq. (2)] to second order in B_1/B_0 .

Turn the page for questions 2 and 3.

2. [33 points] Consider particle of mass m moving in the two-dimensional potential

$$V(x, y) = \begin{cases} Ua^2 \delta(x)\delta(y) & \text{for } |x| \leq a \text{ and } |y| \leq a, \\ +\infty & \text{otherwise,} \end{cases}$$

where a is a positive length and U is an energy scale satisfying $ma^2|U|/\hbar^2 \ll 1$.

Use perturbation theory to calculate the energies of the **six** lowest-lying stationary states, accurate through first order in the small quantity $ma^2|U|/\hbar^2$.

3. A hydrogen atom is in its ground state for times $t < 0$. At $t = 0$, a spatially uniform electric field is suddenly applied to the atom. The form of this field is

$$\mathbf{E}(t) = \hat{\mathbf{z}}E_0 \exp(-t/\tau) \quad \text{for } t \geq 0,$$

where $\hat{\mathbf{z}}$ is a unit vector along the z axis, and E_0 and τ are positive, real quantities.

Focus (as usual) on the quantum-mechanical behavior of the electron (of mass m_e and charge $-e$), treating the atomic nucleus merely as a source of a Coulomb potential. You may ignore the electron's spin. Thus, the unperturbed stationary states of the system are completely specified by three quantum numbers: n , l , and m .

- (a) [10 points] What is the expectation value of the electron's energy immediately after the electric field is switched on?
- (b) [10 points] Specify which unperturbed stationary states have a non-zero occupation probability at times $t > 0$.
- (c) [14 points] Use first-order perturbation theory to estimate the probability that, in the limit $t \rightarrow \infty$, the atom ends up in one of its $n = 2$ levels.