PHY 6646 Spring 2004 – Final Exam

Instructions: Attempt both Question 1 (worth 40 points) and Question 2 (worth 60 points). The maximum score for each part of each question is shown in square brackets. To gain full credit you should explain your reasoning and show all working. Please write neatly and remember to include your name on the front page of your answers.

Please read carefully: During this exam, you may use Shankar's *Principles of Quantum Mechanics* and lecture notes from this course. You may also use standard mathematical tables. You may quote without proof any results given in these sources; however, you should cite the source for the result (e.g., "Shankar page 25"). You are not permitted to consult any other books, notes, or papers, or to communicate with anyone other than the proctor. In accordance with the UF Honor Code, by turning in this exam to be graded, you affirm the following pledge: *On my honor, I have neither given nor received unauthorized aid in doing this assignment.*

1. It should be possible to answer each part of this question with fairly brief mathematical working.

   (a) [16 points] Three identical spinless bosons occupy a three-dimensional, hard-walled cubic box of sides L. At very high temperatures, the system is equally likely to be in any one its low-energy (three-particle) stationary states. Suppose that the energy of the system is measured. Let P_0 be the probability of measuring the ground state energy. Give the probabilities P_1 and P_2 of measuring the first and second excited-state energies, respectively, as multiples of P_0.

   (b) [12 points] A particle of mass m and charge q moves in one-dimension under the influence of the potential $V(x) = V_0 \cosh(x/a)$, where V_0 and a are positive constants. The system is perturbed by the application of a weak electric field of magnitude $\mathcal{E}$, directed along the positive x axis. Without performing any detailed calculation, estimate the shift in the ground-state energy. You should aim to get the correct sign and functional dependence on m, q, V_0, and a, but don't bother to try to evaluate dimensionless numerical prefactors. [Hint: There are a number of different ways to tackle this question. Full credit will be given for any method that yields a reasonable answer.]

   (c) [12 points] A helium atom of mass 7×10^{-27} kg is confined to a rigid, cubic box. Initially, the sides of the box are of length 10 nm, and the atom is in its ground state. Then the walls of the box are moved at a constant speed for a time interval of length T, after which the sides are of length 20 nm. For each of the following cases, estimate the probability that the atom is still in its ground state after the expansion of the box: (i) $T = 10^{-15}$ s; (ii) $T = 10^{-11}$ s; and (iii) $T = 10^{-7}$ s. You may assume that the inner product between the ground state of the initial box and that of the final box is $\langle \psi_0^{(f)} | \psi_0^{(i)} \rangle = (8/9\pi^2)^{3/2}$.

2. This question addresses the problem of "soft sphere scattering" of a point-like particle of mass μ and wave vector k from the spherically symmetric repulsive potential

$$V(r) = \begin{cases} V_0 & \text{for } r < a, \\ 0 & \text{otherwise,} \end{cases}$$

where V_0 and a are positive quantities.

(a) [12 points] Calculate the scattering amplitude $f(\theta)$ using the first Born approximation. Over what range of k should this approximation be valid?

(b) [20 points] Show that for sufficiently small k, the differential scattering cross section takes the form

$$\frac{d\sigma}{d\Omega} = \alpha + \beta \cos\theta + \text{smaller terms.} \tag{1}$$

Provide explicit expressions for α and β. Over what range of k is this form valid?

(c) [16 points] Now consider the partial-wave analysis of the same potential. Show that in the limit of small k, any short-range spherical potential leads to a differential scattering cross section of the form given in Eq. (1).

(d) [12 points] Express the small-k form of the p-wave phase shift δ_1 in terms of α and β entering Eq. (1). Make clear the sign of this phase shift.

Mathematical Formulae

Double angle formulae:

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x,$$

$$\sin 2x = 2\cos x \sin x.$$

The first few Legendre polynomials are

$$P_0(z) = 1, \quad P_1(z) = z, \quad P_2(z) = \frac{1}{2}\left(3z^2 - 1\right), \quad P_3(z) = \frac{1}{2}\left(5z^3 - 3z\right).$$