PHY 6646 Spring 2004 – Homework 3

Due by 5 p.m. on Friday, February 6. No credit will be available for homework submitted after 5 p.m. on Monday, February 9.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. Use a variational wave function $\psi(x) \propto \exp(-\lambda|x|)$ to obtain a variational upper bound on the ground-state energy of the simple harmonic oscillator. Express your final answer as a multiple of the true ground-state energy.

2. Consider a particle of mass m moving in the one-dimensional Gaussian potential

$$V(x) = -V_0 \exp(-\alpha x^2),$$

where V_0 and α are both positive real quantities.

(a) Use the variational method to obtain an upper bound on the energy of the ground state.

(b) Repeat for the first excited state.

3. This question concerns the variational treatment of the energy eigenproblem for the one-dimensional quartic potential

$$V(x) = \lambda x^4, \qquad \lambda > 0.$$

Shankar (p. 430) uses a trial wave function $\psi(x, \alpha) \propto \exp(-\alpha x^2)$ to obtain an upper bound $E(\psi, \alpha_0)$ on the ground-state energy E_0.

(a) Using a one-parameter trial wave function $\phi(x, \beta) \propto x \exp(-\beta x^2)$, find a variational upper bound $E(\phi, \beta_0)$ for the energy E_1 of the first excited state of this potential.

(b) Calculate the energy uncertainty in the state $\psi(x, \alpha)$ [*not* $\phi(x, \beta)$], and hence obtain a lower bound for an energy eigenvalue of the quartic potential. Explain *carefully* the maximum significance that can be attached to this bound.

(c) If you can assume that your value $E(\phi, \beta_0)$ lies within 20% of the true energy E_1, what additional significance can you attach to the lower bound obtained in (b)?

The algebra of this question can be simplified by expressing all lengths in units of $l = (\hbar^2/2m\lambda)^{1/6}$ and all energies in units of $\epsilon = (\hbar^4\lambda/4m^2)^{1/3}$, i.e., by seeking approximate stationary states $\psi(x/l)$ and $\phi(x/l)$ of the operator H/ϵ. (However, you should convert all answers back to the original units.)