PHY 6646 Spring 2004 – Homework 6

Due by 5 p.m. on Monday, March 15. No credit will be available for homework submitted after 5 p.m. on Wednesday, March 18.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. A system has three unperturbed stationary states of energies ϵ_1 (twice) and ϵ_2. In the basis of these stationary states, the perturbed Hamiltonian for this system can be written

$$H = \begin{pmatrix} \epsilon_1 & a & b \\ a & \epsilon_1 & b \\ b & b & \epsilon_2 \end{pmatrix}$$

   where $\epsilon_2 > \epsilon_1$. The quantities a and b are perturbations that are of the same order as each other, and are both small compared to $\epsilon_2 - \epsilon_1$.

   (a) Use Rayleigh-Schrödinger perturbation theory to calculate the energy eigenvalues to second order in the perturbation, and the eigenvectors to first order.

   (b) Use Brillouin-Wigner perturbation theory to calculate the energy eigenvalues to second order in the perturbation, and the eigenvectors to first order.

   Hint: It is advisable first to calculate the eigenvalues to first order. This should help you identify which terms in the implicit eigenvalue equation must be retained in order to get the eigenvalues correct to second order.

2. Shankar Exercise 18.2.1.

3. Shankar Exercise 17.2.4.

   Modification: In part (1), prove the Thomas-Reiche-Kuhn sum rule for an arbitrary potential $V(\mathbf{r})$ in three dimensions (rather than one dimension, as Shankar asks), namely,

$$\sum_m f_{mn}^{(j)} = 1,$$

   where the *oscillator strength* $f_{mn}^{(j)}$ ($j = 1$, 2, or 3, corresponding to x, y, or z, respectively) is defined as

$$f_{mn}^{(j)} = \frac{2m}{\hbar^2}(E_m - E_n)|\langle m|r_j|n\rangle|^2.$$

   Note that the oscillator strength is important for calculating the absorption cross section for electromagnetic radiation.

4. A hydrogen atom initially in its ground state is subjected to an oscillatory electric field directed along the x axis.

   (a) Identify the four states of lowest energy to which first-order transitions can take place.

   (b) Calculate the oscillator strength for each of the transitions identified in (a). What fraction of the total do these oscillator strengths add up to?

   (c) Calculate the integrated cross section for each of the transitions identified in (a). What is the total integrated cross section for all possible electric-dipole transitions out of the ground state of hydrogen when the atom is subjected to an oscillatory electric field directed along the x axis?