PHY 6646 Spring 2004 – Homework 7

Due by 5 p.m. on Monday, March 22. No credit will be available for homework submitted after 5 p.m. on Friday, March 26. (Please note that Kevin Ingersent will be out of town—and unavailable to answer questions about this assignment—from the afternoon of Monday, March 22 until late Thursday, March 25.)

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. Shankar Exercise 18.2.3.

   Addition: Suppose, instead, that the box expands symmetrically to double its size in a time $\tau \gg T$. What is the probability of catching the particle in the ground state of the new box?

2. Shankar Exercise 18.2.5.

3. Sakurai Problem 5.34: Consider a tritium atom made up of an electron and a singly charged ($Z = 1$) nucleus ($^3H^+$). Initially the system is in its ground state ($n = 1, l = 0$). Suppose that the nuclear charge *suddenly increases* by one unit (realistically, by emitting an electron and an antineutrino). This means that the tritium nucleus turns into a helium ($Z = 2$) nucleus of mass 3 ($^3He^+$). Obtain the probability for the system to be found in the ground state of the resulting helium ion. The hydrogenic wave function is given by

$$\psi_{n=1,\,l=0}(\boldsymbol{r}) = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}.$$

4. A one-dimensional harmonic oscillator of mass m, charge q, and angular frequency ω is subjected to a weak electric field E_0, which is spatially uniform and constant in time. Deduce the effective lifetime of the first excited state of the (unperturbed) harmonic oscillator.