PHY 6646 Spring 2004 – Homework 8

Due by 5 p.m. on Friday, April 9. No credit will be available for homework submitted after 5 p.m. on Monday, April 12.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. Shankar Exercise 19.3.2.

2. A particle of mass μ scatters elastically from the spherically symmetric potential $V(r) = V_0 \exp(-r/a)$.

   (a) Use the Born approximation to calculate the differential scattering cross section $d\sigma/d\Omega$ as a function of the scattering angles θ and ϕ.

   (b) From your answer to (a), deduce the total cross section σ.

3. An electron of energy E, initially traveling along the $+z$ direction, scatters elastically off a heavy, neutral atom. Let us represent the atom by a point-like nuclear charge density $+Ze\delta^3(\mathbf{r})$, plus a continuous, spherically symmetric electronic charge distribution $-Ze\rho_e(\mathbf{r})$, where $\int d^3\mathbf{r}\rho_e(\mathbf{r}) = 1$. In this question, we assume that the atom remains stationary, and neglect both the spin of the electron and any effects of particle indistinguishability.

   (a) Write down the potential $V(\mathbf{r})$ that acts on the electron when it is at position $\mathbf{r}$. Do the assumptions that enter the Born approximation hold for this potential?

   (b) Use the Born approximation to calculate the differential scattering cross section for the incident electron in terms of the ion's nuclear and electronic *form factors*, F_n and F_e. The form factor associated with a probability density $\rho(\mathbf{r})$ is

   $$F(\mathbf{q}) = \int d^3\mathbf{r}\rho(\mathbf{r})e^{-i\mathbf{q}\cdot\mathbf{r}},$$

   where $\hbar\mathbf{q}$ is the momentum transfer.

   (c) Evaluate the nuclear form factor $F_n(\mathbf{q})$, and show that for a spherical electronic probability density, the electronic form factor can be written

   $$F_e(\mathbf{q}) = \frac{4\pi}{q}\int_0^\infty dr\,\rho_e(r)\,r\sin qr.$$

4. Shankar Exercise 19.5.4.

5. Shankar Exercise 19.5.5.