

CP Violation

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Three possibilities for the origin of CP violation are discussed: (1) the Standard Model in which all CP violation is due to one parameter in the CKM matrix, (2) the superweak model in which all CP violation is due to new physics and (3) the Standard Model plus new physics. A major goal of B physics is to distinguish these possibilities. CP violation implies time reversal violation (TRV) but direct evidence for TRV is difficult to obtain.

1 Introduction

The symmetries P , C , and T have played a large role in the physics of the past 50 years, where P is left-right symmetry, C is particle-antiparticle symmetry and T for time reversal. Originally, these symmetries were not postulated but discovered theoretically as symmetries of the Hamiltonian of QED. In nuclear and particle physics one tried to guess at the Hamiltonian or Lagrangian density without any classical analogue. The first of these was the Fermi theory of the weak interaction governing nuclear beta decays:

$$\mathcal{H}_W = G_F (\bar{p} \gamma_\mu n \bar{e} \gamma^\mu \nu) \delta(r). \quad (1)$$

Indeed this may be said to mark the beginning of particle physics.

As a result of many experiments on nuclear beta decays it appeared that additional terms would have to be added to Eq. (1) to account for spin-flip (Gamow-Teller) transitions. The major advance, however, followed from the observation in 1956 by Lee and Yang that no experiment had been sensitive to whether parity was conserved in the weak interaction. This led to a number of experiments that showed that P and also C were very much violated. This could be accounted for by replacing the fields in Eq. (1) by left-handed chiral fields¹, or, equivalently, replacing γ_μ by $\gamma_\mu(1 - \gamma_5)$. This became the standard $(V - A)$ theory.

Even before Lee and Yang it had been shown that it was easy to construct Hamiltonians that violated C or P or T but that in fact for every relativistic local quantum field theory one could always define the symmetry CPT . Thus the $(V - A)$ theory violated C and P but still had the symmetries CP and T . Thus it came as a great surprise when it was discovered in 1964 that there was a small violation of CP symmetry in K^0 decays.

The K^0 is characterized by an additive quantum number S conserved in strong and electromagnetic interactions but violated by the weak interaction. Since S is not a good quantum number K^0 with $S = 1$ mixes with \bar{K}^0 with $S = -1$, so one expected the eigenstates to be CP eigenstates

$$\begin{aligned} |K_1\rangle &= \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle), \\ |K_2\rangle &= \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle), \\ |\bar{K}^0\rangle &\equiv CP |K^0\rangle. \end{aligned}$$

The observed eigenstates were K_S (lifetime $\tau_S = 0.9 \times 10^{-10}$ sec) and K_L ($\tau_L = 5.2 \times 10^{-8}$ sec) with $m_L - m_S = 0.48 \Gamma_S \sim 10^{-5}$ eV. The primary decays were

$$\begin{aligned} K_S &\rightarrow \pi^+ \pi^- \text{ and } \pi^0 \pi^0, \\ K_L &\rightarrow 3\pi, \end{aligned}$$

consistent with the CP assignment $K_S = K_1$ and $K_L = K_2$. The discovery in 1964 was that K_L also decayed into $\pi^+ \pi^-$ with a small branching ratio.

There appeared then two alternatives:

1. Modify the $\Delta S = 1$ interaction in some small way from the standard $(V - A)$ form.
2. Assume there exists a much weaker $\Delta S = 2$ interaction that violates CP . This would be described by an effective Hamiltonian (in terms of quarks)

$$\mathcal{H}_{sw} = G_{sw} \bar{s} \mathcal{O} d \bar{s} \mathcal{O} d + h.c., \quad (2)$$

where \mathcal{O} is some Dirac operator. It is sufficient that

$$G_{sw} \sim 10^{-10} \text{ to } 10^{-11} G_F. \quad (3)$$

The superweak idea is that CP violation is confined to $K^0 - \bar{K}^0$ mixing, which is a $\Delta S = 2$ process, second order for the standard theory. In the $K_1 - K_2$ representation

$$M - i \frac{\Gamma}{2} = \begin{pmatrix} M_1 & i m'/2 \\ -i m'/2 & M_2 \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_1 & \\ & \Gamma_2 \end{pmatrix},$$

where m' is the superweak term and the i is required by CPT invariance. Then

$$\begin{aligned} |K_S\rangle &= (|K_1\rangle + \tilde{\varepsilon}|K_1\rangle) / (1 + |\tilde{\varepsilon}|^2), \\ |K_L\rangle &= (|K_2\rangle + \tilde{\varepsilon}|K_2\rangle) / (1 + |\tilde{\varepsilon}|^2), \end{aligned} \quad (4)$$

$$\begin{aligned} \tilde{\varepsilon} &= \frac{-i m'}{(M_1 - M_2) - i(\Gamma_1 - \Gamma_2)/2}, \\ M_1 - M_2 &\approx M_S - M_L = -\Delta M_K, \\ \Gamma_1 - \Gamma_2 &\approx \Gamma_S - \Gamma_L \approx \Gamma_S. \end{aligned} \quad (5)$$

The observations give $\tilde{\varepsilon} \simeq 2 \times 10^{-3}$ which then determines m' from which Eq. (3) follows.

The superweak theory made three predictions:

1. The CP violation is completely described by $\tilde{\varepsilon}$; in particular the observables

$$\eta_{+-} = \eta_{00} = \tilde{\varepsilon},$$

where

$$\begin{aligned} \eta_{+-} &= A(K_L \rightarrow \pi^+ \pi^-) / A(K_S \rightarrow \pi^+ \pi^-), \\ \eta_{00} &= A(K_L \rightarrow \pi^0 \pi^0) / A(K_S \rightarrow \pi^0 \pi^0). \end{aligned}$$

2. The phases of η_{+-} and η_{00} are equal and determined by Eq. (5) to be about 43.5° using the empirical values of ΔM_K and Γ_S .
3. The CP violation is so small that it will not be seen anywhere else.

These predictions proved all too true for more than 25 years.

The alternative to superweak says that CP is violated in the decay amplitude $A_0 e^{i\delta_0}$ and $A_2 e^{i\delta_2}$ corresponding to final $I = 0$ and $I = 2$ $\pi\pi$ states, where δ_I is the strong phase shift. From CPT and unitarity $\text{Im } A_I$ give the amplitudes for the CP -violating transitions $K_2 \rightarrow \pi\pi$. Since there is still in any model the contribution m' there are now three CP -violating quantities²

$$m', \text{Im } A_0, \text{Im } A_2.$$

However there is a choice of phase convention using the $U(1)$ transformation $s \rightarrow s e^{-i\alpha}$ under which the strong and electromagnetic interactions are invariant. For the infinitesimal $U(1)$ as an example

$$\begin{aligned} s &\rightarrow s(1 - i\alpha), \\ \text{Im } A_I &\rightarrow \text{Im } A_I - \alpha \text{Re } A_I, \\ m' &\rightarrow m' + \alpha(M_1 - M_2). \end{aligned}$$

Thus there are only two independent quantities which may be chosen as

$$\begin{aligned}\varepsilon' &\propto \text{Im } A_0/\text{Re } A_0 - \text{Im } A_2/\text{Re } A_2, \\ \varepsilon &\simeq \tilde{\varepsilon} + i(\text{Im } A_0/\text{Re } A_0).\end{aligned}\tag{6}$$

Then

$$\begin{aligned}\eta_{+-} &= \varepsilon + \frac{\varepsilon'}{1 + \omega/\sqrt{2}}, \\ \eta_{00} &= \varepsilon - \frac{2\varepsilon'}{1 + \sqrt{2}\omega},\end{aligned}\tag{7}$$

where $\omega = \text{Re } A_2/\text{Re } A_0 \simeq 0.045$ and the last numerical result is empirical and a sign of the $\Delta I = 1/2$ rule. The quantity ε' is a measure of CP violation in the decay amplitudes and so not due to a superweak interaction.

2 The Standard Model vs Superweak

For 35 years experiments have sought to determine ε' by finding the difference between η_{+-} and η_{00} and thus detecting CP violation in the decay amplitude. The experiments actually measure

$$\left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 \simeq 1 + 6 \text{Re}(\varepsilon'/\varepsilon).\tag{8}$$

By chance the phase of ε given to a good approximation by Eq. (5) and the phase of ε' given by $(\pi/2 - \delta_0 + \delta_2)$ are both approximately equal to 45° so that $\text{Re}(\varepsilon'/\varepsilon) \simeq |\varepsilon'/\varepsilon|$. Recent experiments have left no doubt that $|\varepsilon'/\varepsilon|$ is non-zero and my average of the results is

$$|\varepsilon'/\varepsilon| = (20 \pm 6) \times 10^{-3}.$$

The big development in weak interactions was the spontaneously broken gauge theory. Originally given by Weinberg as a theory of leptons it was extended by Glashow, Iliopoulos and Maiani (GIM) to quarks by adding a fourth quark charm. With the discovery of neutral currents in 1973 this became the new Standard Model. A striking feature of this new theory was that it had CP and T invariance and so provided no solution to the CP violation problem. In one paragraph of a paper³ in 1973 that few people noticed for several years it was proposed that if there were six quarks then CP violation could be allowed in the Standard Model. With the discovery of b quark physics in 1978 this became the standard CKM model of CP violation.

The only place one can insert complex phases to give CP violation in the Standard Model is in the Yukawa interaction. This shows up after symmetry

breaking in the unitary CKM matrix V_{ji} connecting down quarks d_i to up quarks u_j in the interaction with the W bosons:

$$\bar{u}_j V_{ji} \gamma^\mu (1 - \gamma_5) d_i W_\mu. \quad (9)$$

With only four quarks the unitary 2×2 matrix has three phases which can be removed by the $U(1)$ transformations of S , charm C , and electric charge Q . With six quarks the 3×3 matrix has six phases but only five can be removed. In a standard phase convention there are two matrix elements with large phases:

$$\begin{aligned} V_{ub} &= |V_{ub}| e^{-i\gamma} \simeq A \lambda^3 (\rho - i\eta), \\ V_{td} &= |V_{td}| e^{-i\beta} \simeq A \lambda^3 (1 - \rho - i\eta), \end{aligned} \quad (10)$$

where the last equality corresponds to a standard parameterization in powers of $\lambda = \sin \theta_c \simeq 0.22$ (see the lectures by Falk).

The CP violation is directly proportional to η . The CP -violating observable ε (determined from η_{+-}) is calculated from the box diagram giving the $K - \bar{K}$ mixing parameter m' . (Note $\text{Im} A_0 / \text{Re} A_0$ is very small compared to $\tilde{\varepsilon}$ in using Eq. (6).) This is directly proportional to η and requires that η be between 0.2 and 0.6. This is very consistent with the constraint from the magnitude of $|V_{ub}|$. Note this consistency is significant; when one thought m_t was 20 GeV instead of 170 GeV the value of η required was larger and the consistency was much less obvious. Given this value of η the phase γ is expected to lie between 45° and 135° and the phase β between 15° and 30° .

The calculation of ε' in the Standard Model is very uncertain due to the difficulty of calculating hadronic matrix elements (see the lectures by Buchalla). The dominant contribution is the so-called penguin diagram in which a loop involving t or c quarks emits a gluon. The $s \rightarrow d + g$ transition contributes only to $\text{Im} A_0$. There is also an electroweak penguin $s \rightarrow d + (\gamma \text{ or } Z)$ which can contribute to $\text{Im} A_2$ and somewhat decreases ε' (see Eq. (6)). Different theoretical calculations give values of ε'/ε from 1.3 to $6 \times 10^{-3} \eta$ with large errors. Given the uncertainties it seems the Standard Model can explain, though not really predict, the value of ε'/ε .

The direct CP violation indicated by ε'/ε is a major blow to the superweak theory. Before completely abandoning superweak I thought it would be good to define it. There are many possible theories which are effectively superweak; their common feature is that at a low energy scale their effective interaction is given by

$$\mathcal{H}_{eff} = \mathcal{H}_0 + \mathcal{H}_{sw},$$

where \mathcal{H}_0 is the Standard Model with the CP -violating parameter η chosen to

zero so that all CP violation is given by \mathcal{H}_{sw} which contains terms of the form

$$G_{ijkl} \bar{q}_i q_j \bar{q}_k q_l$$

with all $G_{ijkl} \ll G_F$. In particular, as noted, to explain ε , $G_{\bar{s}d\bar{s}d} \sim 10^{-10}$ to $10^{-11} G_F$. In general, the terms in \mathcal{H}_{sw} are derived from diagrams involving heavy particles beyond those in the Standard Model⁴.

Because ε' is only about 5×10^{-6} the possibility arises that in some theories that are effectively superweak such a value of ε' could be explained. In fact as long as one is only dealing with the first four quarks one can think of the Standard Model as effectively superweak. Then \mathcal{H}_0 is the 4-quark Standard Model, which, as we noted, is CP -invariant and \mathcal{H}_{sw} corresponds to effective CP -violating interactions due to diagrams involving the heavy t quark which determines ε and ε' .

The really distinctive features of the CKM model appear only when we consider the CP violation involving B mesons. The $B - \bar{B}$ mixing is dominated by the box diagram involving t quarks giving in the $B - \bar{B}$ representation

$$M_{12} \propto (V_{td}^* V_{tb})^2 \simeq A^2 \lambda^6 [(1 - \rho)^2 + \eta^2] e^{2i\beta}.$$

Thus in this phase convention there is large CP violation in the mixing. The time-dependent $B^0(\bar{B}^0)$ decay rate to a CP eigenstate has a term⁵

$$\pm \eta_f \sin 2(\beta - \varphi_f) \sin \Delta m t, \quad (11)$$

where φ_f is the CP -violating phase in the decay amplitude, η_f is the CP eigenvalue, and $+(-)$ corresponds to initial $B^0(\bar{B}^0)$. The observation of this asymmetry is the major goal of B factories. The first example will be the decay $B^0(\bar{B}^0) \rightarrow \Psi K_S$ which is due to the transition $b \rightarrow c \bar{c} s$, for which $\varphi_f = 0$. A result from the hadron collider experiment CDF has given the result $\sin 2\beta = 0.8 \pm 0.4$. By the time these lectures are published more accurate values should be available from the BABAR and BELLE experiments.

Large positive value of $\sin 2\beta$ will give strong qualitative evidence for the Standard Model. Nevertheless this asymmetry can still be blamed on superweak mixing with $G_{\bar{b}d\bar{b}d} \sim 10^{-7}$, although only in special models would one expect such a large effect in $B - \bar{B}$ mixing. To finally kill all superweaklike theories one must look for the expected large CP violation in the B decay amplitude. To allow for the superweak possibility we call the result of the first experiment $\sin 2\tilde{\beta}$; then if a different decay to a final state f' gives the asymmetry $\sin 2(\tilde{\beta} - \varphi_{f'})$, the phase $\varphi_{f'}$ is the relative phase of $B \rightarrow f'$ to that of $B \rightarrow \Psi K_S$.

The most obvious choice for f' is the decay $B \rightarrow \pi^+ \pi^-$. In the “tree” approximation this is due to $b \rightarrow u \bar{u} d$ and so that the phase $\varphi_{f'} = \varphi_\pi = -\gamma$. For example, if $\gamma = 90^\circ$ then $\sin 2(\tilde{\beta} + \gamma) = -\sin 2\tilde{\beta}$, a very large direct CP -violating effect. This I call the ε' experiment for the B system. In contrast to ε' for the K system which is 5×10^{-6} we expect here $\sin 2(\tilde{\beta} - \varphi_\pi) - \sin 2\tilde{\beta}$ of order unity!

However, this may not prove so easy. The branching ratio $B^0 \rightarrow \pi^+ \pi^-$ may be only about 5×10^{-6} . Furthermore if $\varphi_\pi = 2\tilde{\beta} - \pi/2$ then $\sin 2(\tilde{\beta} - \varphi_\pi) = \sin 2\tilde{\beta}$ by accident. In the tree approximation if $2\tilde{\beta} = 45^\circ$ this would occur for $\gamma = 45^\circ$ near the end of the allowed region. In fact, however, one expects a large contribution from “penguin” diagrams corresponding to $b \rightarrow d + gluon$ via a t quark loop (see the lectures of Rosner). Thus, the decay amplitude is given as

$$A(B \rightarrow \pi^+ \pi^-) = T e^{-i\gamma} + P e^{i\tilde{\beta}} e^{i\Delta}, \quad (12)$$

since the penguin is proportional to $V_{tb} V_{td}^*$ and so has the phase β . Estimates based on the rate of $B \rightarrow K \pi^6$ suggest P/T as high as 0.4. Assuming the strong phase Δ is small, then $|\varphi_\pi|$ is less than $|\gamma|$ so that $\varphi_{f'} = -45^\circ$ corresponds to $\gamma \sim 65^\circ$. For values of $\gamma < 75^\circ$ it may be difficult to detect the difference between $\sin 2\tilde{\beta}$ and $\sin 2(\tilde{\beta} - \varphi_\pi)$; since present evidence from the ratio of $\Delta m(B_s)/\Delta m(B_d)$ suggests $\gamma < 90^\circ$, the region $\gamma < 75^\circ$ corresponds to most of the range of γ . It may prove easier then to consider other decays such as $B^0 \rightarrow \rho^+ \pi^-$ and $\rho^- \pi^+$.

An alternative would be to consider decays that are dominated by the $b \rightarrow d$ penguin. In this case $\varphi_f = \beta$ and $\sin 2(\tilde{\beta} - \varphi_f) = \sin 2(\tilde{\beta} - \beta)$. In the Standard Model $\tilde{\beta} = \beta$ and so there is no asymmetry. The observation of such a zero asymmetry in contrast to the large asymmetry for $B^0 \rightarrow \Psi K_S$ would then be evidence for a large direct CP violation. Unfortunately the best candidates for such decays might not be very practical; they correspond to $b \rightarrow d \bar{s} s$ yielding $B^0 \rightarrow K_S K_S$, $B^0 \rightarrow \rho^0 \eta$, etc. Note that even if the decay were not pure penguin one would expect an asymmetry very different from that for $B^0 \rightarrow \Psi K_S$.

Another possibility is a CP -violating effect in which mixing is not involved. One can look for a difference between the rates of $B^+ \rightarrow f$ and $B^- \rightarrow \bar{f}$ or in the case of B^0 looking at time $t = 0$ before mixing for the difference between $B^0 \rightarrow f$ and $\bar{B}^0 \rightarrow \bar{f}$. In practice this means looking for the $\cos \Delta m_B t$ term⁵ rather than the $\sin \Delta m_B t$. As an example, consider $B^0(\bar{B}^0) \rightarrow \pi^+ \pi^-$; from Eq. (12)

$$A_\pi = \frac{\Gamma(B^0 \rightarrow \pi^+ \pi^-) - \Gamma(\bar{B}^0 \rightarrow \pi^+ \pi^-)}{\Gamma(B^0 \rightarrow \pi^+ \pi^-) + \Gamma(\bar{B}^0 \rightarrow \pi^+ \pi^-)},$$

$$= -\frac{2TP \sin(\beta + \gamma) \sin \Delta}{T^2 + P^2 + 2TP \cos(\beta + \gamma) \cos \Delta}. \quad (13)$$

For $\beta + \gamma \simeq 90^\circ$ this gives (with $P/T \equiv r$)

$$A_\pi = -\frac{2r}{1+r^2} \sin \Delta.$$

Thus a very large CP -violating asymmetry is possible if r is greater than 0.3, but it all depends on the strong phase Δ .

It is very difficult to make definite statements about the strong phases in B decays in contrast to K decays. For $K \rightarrow \pi\pi$ the final state interaction can be thought of as elastic scattering with phase shifts δ_2 and δ_0 corresponding to $\pi\pi$ states with $I = 2$ and $I = 0$. However, $\pi\pi$ s -wave scattering at 5 GeV is highly inelastic involving many channels. The phase Δ arises from the absorptive parts of diagrams corresponding to the strong scattering from other final states into the $\pi\pi$ state. For any weak interaction operator \mathcal{O}_i we can define the real decay amplitude in lowest order

$$M_{fi} = M_{fi}^0 = \langle f | \mathcal{O}_i | B \rangle.$$

If f were an eigenstate one would then multiply this by $e^{i\delta_f}$. Going from the eigenstate basis to the states of interest

$$M_{fi} = \sum_{f'} \langle f | S^{1/2} | f' \rangle \langle f' | \mathcal{O}_i | B \rangle, \quad (14)$$

where S is the strong-interaction S matrix.

The sum in Eq. (14) is over a large number (almost uncountable) of states. One can only make some general comments about it:

1. The strong phase depends on the operator \mathcal{O}_i that affects the relative importance of different states f' . The phase Δ in Eqs. (12) and (13) is the difference between the strong phase of the “tree” operator and that of the “penguin”.
2. Since the strong scattering is expected to be very inelastic the diagonal element $\langle f | S^{1/2} | f \rangle$ has as its major effect the reduction of M_{fi} ; this is a kind of absorption effect. Thus we could write

$$M_{fi} = M_{fi}^0 a_i + i R_i = |M_{fi}| e^{i\delta_i},$$

where $a < 1$ is the reduction due to absorption. For a “typical” state, by unitarity, the scattering “in” due to R compensates for the scattering “out” so

$$R_i = \sqrt{1 - a_i^2} = \sin \delta_i. \quad (15)$$

3. An estimate can be based on a crude statistical argument⁷ in which case one can reduce the multichannel problem to an equivalent 2-channel problem

$$S = \begin{pmatrix} \cos 2\theta & i \sin 2\theta \\ i \sin 2\theta & \cos 2\theta \end{pmatrix}.$$

For $f = \pi\pi$ the diagonal element $\cos 2\theta$ can be estimated by extrapolating data from πN scattering to $\pi\pi$ giving

$$\cos 2\theta = \eta \simeq 0.7.$$

Then Eq. (14) becomes

$$\begin{aligned} M_{1i} &= \cos\theta A_{1i} + i \sin\theta A_{2i}, \\ \tan\delta_i &= i \tan\theta \frac{A_{2i}}{A_{1i}}, \end{aligned} \tag{16}$$

where $\tan\theta = \left(\frac{1-\cos 2\theta}{1+\cos 2\theta}\right)^{1/2} \simeq 0.42$.

The “typical” result corresponds to $A_2 = A_1$ and gives a strong phase of 20° to 25° . The quantitative conclusion from Eq. (16) is that if the state of interest (labeled 1 here) is a “more probable” final state than the states into which that state scatters (lumped into state 2 here) then the strong phase may be small.

In conclusion, after it is clearly shown that CP violation is not superweak the next step is to find quantitative tests of the Standard Model by showing the consistency of a number of different experiments. This is a major program for the next decade.

3 Time Reversal Violation

By the CPT theorem CP violation implies time-reversal violation (TRV). Strong evidence for CPT invariance comes from the phase of ε determined from Eq. (5). CPT violation would allow a real off-diagonal term m'' in the matrix in addition to im' and thus would change the phase. Since the measured phase agrees with theory to about 1° there is a strong limit on m'' which corresponds to a limit on $[m(K^0) - m(\bar{K}^0)]/m_K \leq 10^{-18}$. Nevertheless, it is of great interest to look for direct evidence for TRV both as another way to study CP violation as well as a way to demonstrate T violation in a straightforward way⁸.

We discuss here four types of direct evidence for TRV; by this I mean a single experiment that by itself is seen to violate T . These are

1. A non-zero value of a T -odd observable in a stationary state. The simplest example is the electric dipole moment of an elementary particle or an atom.
2. A violation of the reciprocity condition on the S matrix

$$S_{fi} = S_{-i,-f}$$

corresponding to comparing a reaction and its inverse.

3. A non-zero value of a T -odd observable in the final state of a weak decay. As discussed below this depends upon the neglect of final-state interactions.
4. In an oscillation a difference in the probability of $a \rightarrow b$ from $b \rightarrow a$ at a given time. It is interesting to note that each example immediately implies a test of CP violation (conceptual if not practical) by going to the anti-particles. In contrast the simplest tests of CP violation have no direct relation to TRV; for example, $\Gamma(B \rightarrow f) = \Gamma(\bar{B} \rightarrow \bar{f})$ involves a rate which has nothing to do with a TRV observable.

Experimental limits on the dipole moments of the electron and neutron are

$$\begin{aligned} d_n &\leq 10^{-25} \text{ e - cm,} \\ d_e &< 4 \times 10^{-27} \text{ e - cm.} \end{aligned}$$

In the Standard Model d_n is second-order weak and the calculation depends on long-distance effects giving of order 10^{-32} e-cm; d_e is third order and perhaps 10^{-38} e-cm. Thus the interest lies in the search for physics beyond the Standard Model (see the lecture by Thomas).

Many tests of the reciprocity relations exist for strong interactions although they are of limited accuracy; it is very hard to study the reverse of weak interactions. An interesting proposal by Bowman involves the scattering of slow neutrons from polarized nuclei in the resonance region. One compares the observable $\langle \vec{\sigma}_n \cdot \vec{I} \times \vec{k} \rangle$ for incident polarization $\vec{\sigma}_n$ and final polarization $\vec{\sigma}'_n$, where \vec{I} is the nuclear polarization. T -violating effects in the nuclear wave functions could enhance the result.

An example of a “ T -odd observable” in the final state of decay is the muon polarization

$$P = \langle \vec{\sigma}^\mu \cdot \vec{k}_\mu \times \vec{k}_\nu \rangle$$

in the decay $K \rightarrow \pi + \mu + \nu$. This does not really violate T except in the Born approximation when final state interactions (FSI) can be avoided. As a simple didactic example, consider the scattering from a potential

$$V_0 + V_1 \vec{\sigma} \cdot \vec{L}$$

which certainly is T -invariant. The resulting amplitude is

$$A = f_0 + i f_1 \vec{\sigma} \cdot \hat{n}$$

where \hat{n} is the normal to the scattering plane. In the Born approximation f_0 and f_1 are real and so $\langle \vec{\sigma} \cdot \hat{n} \rangle = 0$, but beyond the Born approximation f_0 and f_1 are complex; for example, if s and p waves dominate f_0 would have a phase $e^{i\delta_0}$ and f_1 the phase $e^{i\delta_1}$.

For the case of $K^0 \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu$ there is a Coulomb FSI so that without T violation, $P \sim 10^{-3}$; for the case of $K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu$ the FSI involves 2γ exchange so that $P \sim 10^{-6}$. In the Standard Model the real TRV is expected to vanish in semi-leptonic decays. One would expect a real TRV in non-leptonic decays such as $\Lambda \rightarrow p \pi^-$ where there is a defined parameter

$$\beta \propto \langle \vec{\sigma}^\Lambda \cdot \vec{\sigma}^p \times \vec{k} \rangle .$$

However, the FSI effect is much larger being proportional to $\sin(\delta_p - \delta_s)$ where δ_p, δ_s are the π - p phase shifts. If the experiment is also done with $\bar{\Lambda}$, then $\beta + \bar{\beta}$ is a clear CP -violating effect and is associated with true TRV, but this is hardly “direct evidence” of TRV.

A large “ T -odd observable” has been found in the decay $K_L \rightarrow \pi^+ \pi^- e^+ e^-$

$$C = \langle \hat{n}_e \times \hat{n}_\pi \cdot \hat{z} \rangle \langle \hat{n}_e \cdot \hat{n}_\pi \rangle ,$$

where $\hat{n}_e(\hat{n}_\pi)$ are the normals to the $e^+ e^-(\pi^+ \pi^-)$ planes and \hat{z} is the unit vector between the pairs. This was predicted as a result of $K - \bar{K}$ mixing as an interference between an $M1$ virtual γ from $K_2 \rightarrow \pi \pi \gamma$ and an $E1$ virtual bremsstrahlung from $K_1 \rightarrow \pi \pi \gamma$. The theoretical result⁹ is

$$C = 0.15 \sin(\varphi_\varepsilon + \Delta),$$

where $\Delta \simeq 30^\circ$ comes from $\pi \pi$ phase shifts. The experimental result¹⁰ verifies this; the result is so large because for the $e^+ e^-$ energy considered the $E1$ is much larger than $M1$ which compensates for the small admixture $|\varepsilon|$ of K_1 . Since Δ is involved this is not again obvious TRV. It is of didactic interest to consider the limit $\Delta \rightarrow 0$. In this case C is proportional to $\sin \varphi_\varepsilon$. We know

that $\varphi_\varepsilon \simeq \frac{\pi}{4}$ in accordance with CPT invariance and T violation. If we had assumed CPT violation and T invariance it follows from an analysis like that of Eq. (5), replacing im' by m'' , that $\varphi_\varepsilon \simeq \frac{3\pi}{4}$ and so we get the same value of C even though there is no TRV and no FSI!

The explanation lies in the fact that we are here sensitive to higher order weak effects which show up in $\Delta\Gamma$ in Eq. (5). I call this the “initial state interaction”. This could be considered as a contribution to the decay amplitude of the form

$$K^0 \rightarrow \pi^+ \pi^- \rightarrow \bar{K}^0 \rightarrow \pi^+ \pi^- e^+ e^-$$

with $\pi^+ \pi^-$ on the mass shell thus giving an absorptive part. Only if $\Delta = 0$ and $\Delta\Gamma = 0$ would a non-zero C directly show TRV.

The possibility of seeing CP violation in neutrino oscillations from the difference between $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ has been discussed in many papers (see the talk by Kayser). The same formula gives the TRV difference between $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$. Note that the time dependence (or, equivalently, the distance dependence) of the difference is an odd function of time. The possibility of doing the TRV experiment requires beams of both ν_μ and ν_e as has been proposed for “neutrino factories” based on a muon storage ring.

In the CP LEAR experiment¹¹ a difference has been observed between the transitions $K^0 \rightarrow \bar{K}^0$ and $\bar{K}^0 \rightarrow K^0$. Here the initial K^0 (\bar{K}^0) has been identified by its associated production with a K^+ (K^-) and the final \bar{K}^0 (K^0) by the charge of the lepton in the semi-leptonic decay. The result in agreement with a simple calculation is

$$\frac{\Gamma(K^0 \rightarrow \bar{K}^0) - \Gamma(\bar{K}^0 \rightarrow K^0)}{\Gamma(K^0 \rightarrow \bar{K}^0) + \Gamma(\bar{K}^0 \rightarrow K^0)} = 4 \operatorname{Re} \varepsilon$$

independent of time. This seems somewhat strange since we expected an odd function of time. One can also ask from unitarity if K^0 goes to \bar{K}^0 more than \bar{K}^0 goes to K^0 what compensates for this. The answer is that the \bar{K}^0 decays to $\pi\pi$ more than K^0 . Thus, decay plays an essential role, rendering this as a direct test of TRV somewhat questionable.

As we have noted the phase of ε is completely consistent with CPT invariance. There is no reason to doubt CPT invariance, which appears very fundamental, and so we conclude that the observed CP violation is associated with TRV. Nevertheless, unambiguous direct tests of TRV may prove very difficult.

Acknowledgments

Many of the early papers on CP violation are reprinted in L. Wolfenstein, *CP Violation* (North Holland) (1989). This work was supported by the U.S. Department of Energy under Grant No. DE-FG02-91ER40682.

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