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TWO-LOOP RENORMALIZATION GROUP EQUATIONS IN A GENERAL QUANTUM FIELD THEORY I. WAVE FUNCTION RENORMALIZATION

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Abstract

The two-loop renormalization group equations in a general renormalizable field theory with scalar, spin 1/2, and (vector) gauge fields are considered. In this paper, the anomalous dimensions associated with the wave function renormalizations of the fields are computed in a general R_ξ gauge. The wave function renormalization of the gauge field in background field gauge, and hence the two-loop β-function for the gauge coupling, are also evaluated.

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1. Introduction

Analysis based on the renormalization group [1] (for a recent review, see [2]) has played an important role in the development of quantum chromodynamics (for reviews, see [3]) and grand unified gauge theories ([4]; for review, see [5]), in addition to finding useful application in the theory of critical phenomena and phase transitions (see, for example, [6]).

Computation of the coefficients in the renormalization group (RG) equations for Green functions has been done in perturbation theory for a variety of renormalizable quantum field theories. The β-function for the gauge coupling in quantum chromodynamics (QCD) with fermions has been carried out to one-loop [7], two-loop [8], and, more recently, to three-loop order [9]. Anomalous dimensions of various operators have been computed to one-loop [10] and two-loop [11] order by several groups (see also recent reviews [3]).

In a scalar field theory with quartic self-coupling, a series of computations [12] has led to the evaluation of the anomalous dimension of the scalar field through five-loop order, and the β-function of the scalar quartic coupling through four-loop order. For a spin 1/2 fermion coupled to the scalar through a pseudoscalar Yukawa coupling, β-functions and anomalous dimensions have been evaluated through two-loop order [13]. For a summary of various results, see [14].

For a general renormalizable field theory with scalars, spin 1/2 fermions, and vector gauge fields, the evolution equations for the coupling constants have been given to one-loop order by Cheng, Eichten, and Li [15], but only fragmentary extensions to two-loop order exist. In a series of papers, of which this is the first, we present a complete analysis to two-loop order of the evolution equations for the coupling constants and of the anomalous dimensions of the field operators in such a general theory.

In this paper, we derive the anomalous dimensions of the scalar, fermion, and gauge field operators. By working in background field gauge (see [16] and references contained therein), we then obtain the two-loop β-function for the gauge coupling, including the contribution of the Yukawa couplings (the scalar quartic couplings do not contribute to the β-function of the gauge coupling until three-loop order). The corresponding two-loop β-functions for the Yukawa and scalar quartic couplings will be given in subsequent papers.

In Section 2, we establish our notation, and give the formal analysis which allows us to extract the anomalous dimensions from the divergent parts of the relevant two-loop diagrams. In Sections 3-5, we present our results for the anomalous dimensions of the scalar, fermion, and gauge field operators, respectively. In Section 6, we discuss our results, and compare them with partial results in the existing literature [11-14, 17]. In Appendix A, we collect some useful group theoretic results. In Appendix B, we give explicit results for the β-functions for the gauge couplings in SU(3) x SU(2) x U(1) including the effects of Yukawa couplings.

2. Formal Analysis

We consider a general renormalizable field theory with vector gauge fields V_μ^A associated with a compact simple (or semi-simple) gauge group G, scalar fields ϕ_a, and two-component spinor fields ψ_j. The representations (reducible, in general) of G under which the scalar and spinor fields transform will be denoted by S and F, respectively.

The gauge-invariant Lagrange density of the theory has the general form

$$\begin{aligned}\mathscr{L} = & -\frac{1}{4} F^A_{\mu\nu} F^A_{\mu\nu} + \frac{1}{2} D_\mu\phi_a D_\mu\phi_a \\ & + i\, \psi_j^\dagger \sigma_\mu D_\mu \psi_j - (Y^a_{jk} \psi_j \zeta \psi_k \phi_a + \text{H.C.}) \\ & - \frac{1}{4!} \lambda_{abcd}\, \phi_a\phi_b\phi_c\phi_d \\ & + \cdots \text{ (mass terms)} \\ & + \cdots \text{ (gauge-fixing + ghost terms)}\end{aligned} \tag{2.1}$$

where the mass terms are not relevant to the present calculation, and the gauge-fixing and ghost terms are here those appropriate to a standard R_ξ gauge [18,19], or to the corresponding background field gauge [16]. Here

$$F^A_{\mu\nu} = \partial_\mu V^A_\nu - \partial_\nu V^A_\mu + g f^{ABC} V^B_\mu V^C_\nu \tag{2.2}$$

where the f^{ABC} are the structure constants of the gauge group, and g is the gauge coupling constant (if the gauge group is only semi-simple, there are modifications which are explained in Appendix A). Also,

$$D_\mu\phi_a = \partial_\mu\phi_a + i g\, \theta^A_{ab} V^A_\mu \phi_b \tag{2.3}$$

$$D_\mu\psi_j = \partial_\mu\psi_j + i g\, t^A_{jk} V^A_\mu \psi_k \tag{2.4}$$

where $\underset{\sim}{\theta}^A \equiv (\theta^A_{ab})$ and $\underset{\sim}{t}^A \equiv (t^A_{jk})$ are the (Hermitian) representation matrices for the generators of the gauge group acting on the scalar and fermion fields, respectively. (We implicitly assume a real representation for the scalars, so that the $\underset{\sim}{\theta}^A$ are imaginary and antisymmetric.) $\zeta = \pm i\sigma_2$ is the spinor metric, and, in general, our notation for two-component spinors follows that of Sikivie and Gürsey [20]. The constraints imposed on the Yukawa coupling matrices

$$\underset{\sim}{Y}^a \equiv (Y^a_{jk}) \tag{2.5}$$

by invariance under the gauge group G are described in Appendix A.

The renormalization group equations characterize the response of the Green functions of the theory to a change in the scale parameter μ, with dimension of mass, which is necessarily introduced in the renormalization of the theory. The coefficients in these equations are associated with the finite change induced by a change of this scale parameter in the infinite wave function, mass, and vertex renormalization constants of the theory (for detailed discussion, see [2,3] and references given in those reviews).

In this paper, we calculate the wave function renormalization constants of the fields (to two-loop order in perturbation theory) using dimensional regularization [21] and a mass-independent renormalization scheme [22,23] based on the modified minimal subtraction ($\overline{MS}$) algorithm [24].

In this scheme, the renormalized coupling constants in $d \equiv 4-2\varepsilon$ dimensions, denoted by x_k, are related to the corresponding bare coupling constants x_{kB} by an expansion of the form

$$x_{kB}\,\mu^{-\rho_k\varepsilon} = x_k + \sum_{n=1}^{\infty} a_k^{(n)}(x)\,\frac{1}{\varepsilon^n} \tag{2.6}$$

where μ is the scale parameter with dimension of mass which here must be introduced in order to define a dimensionless renormalized coupling constant, and $\rho_k = 1$ for gauge and Yukawa coupling constants, $\rho_k = 2$ for scalar quartic coupling constants. The coefficients $a_k^{(n)}(x)$ are to be computed (in practice, in perturbation theory).

The bare coupling constants are independent of renormalization scale, but the renormalized coupling constants will depend on the choice of the scale parameter μ, and the β-functions

$$\beta_k(x) \equiv \mu \left.\frac{dx_k}{d\mu}\right|_{\varepsilon=0} \tag{2.7}$$

which appear in the RG equations are derived by observing that Eq. (2.6) implies

$$\begin{aligned} -\rho_k\,\varepsilon \left\{x_k + \sum_{n=1}^{\infty} a_k^{(n)}(x)\,\frac{1}{\varepsilon^n}\right\} &= \\ = \mu\frac{dx_k}{d\mu} + \sum_{n=1}^{\infty}\sum_{\ell} \mu\,\frac{dx_\ell}{d\mu}\,\frac{\partial a_k^{(n)}}{\partial x_\ell}\,\frac{1}{\varepsilon^n}\,. \end{aligned} \tag{2.8}$$

Collecting coefficients of like powers of ε then gives

$$\mu\frac{dx_k}{d\mu} = \beta_k(x) - \rho_k x_k \varepsilon \tag{2.9}$$

for all ε,

$$\beta_k(x) = \sum_{\ell} \rho_\ell x_\ell \frac{\partial a_k^{(1)}}{\partial x_\ell} - \rho_k a_k^{(1)}(x) \tag{2.10}$$

whence the β-functions are completely determined by the coefficients of the single pole term in the expansion (2.6), and

$$a_k^{(n+1)}(x) = \sum_{\ell}\left\{\beta_\ell \frac{\partial a_k^{(n)}}{\partial x_\ell} - \rho_\ell x_\ell \frac{\partial a_k^{(n+1)}}{\partial x_\ell}\right\} \tag{2.11}$$

($n = 1,2,\ldots$) whence the residues of the higher-order poles in (2.6) are completely determined by those of the lower-order pole--this provides a useful computational check.

The wave function renormalization constant Z_Γ associated with an irreducible self-energy part Γ has the expansion

$$Z_\Gamma = 1 + \sum_{n=1}^{\infty} C_\Gamma^{(n)}(x)\,\frac{1}{\varepsilon^n} \tag{2.12}$$

and the anomalous dimension for the corresponding field is defined by

$$\gamma_\Gamma = \frac{1}{2}\,\mu\,\frac{d}{d\mu}\,\ell n\, Z_\Gamma . \tag{2.13}$$

Then

$$\gamma_\Gamma Z_\Gamma = \frac{1}{2}\,\mu\,\frac{dZ_\Gamma}{d\mu} = \frac{1}{2}\sum_{n=1}^{\infty}\sum_{\ell}\mu\,\frac{dx_\ell}{d\mu}\,\frac{\partial C_\Gamma^{(n)}}{\partial x_\ell}\,\frac{1}{\varepsilon^n} \tag{2.14}$$

and, inserting the expansion (2.12) into (2.14) and using (2.9), there follows

$$\gamma_\Gamma = -\frac{1}{2}\sum_{\ell}\rho_\ell x_\ell\,\frac{\partial C_\Gamma^{(1)}}{\partial x_\ell} \tag{2.15}$$

whence the anomalous dimension is determined from the single pole term in (2.12), and

$$\gamma_\Gamma C_\Gamma^{(n)}(x) = \frac{1}{2}\sum_{\ell}\left\{\beta_\ell\,\frac{\partial C_\Gamma^{(n)}}{\partial x_\ell} - \rho_\ell x_\ell\,\frac{\partial C_\Gamma^{(n+1)}}{\partial x_\ell}\right\} \tag{2.16}$$

($n = 1,2,\ldots$) which provides a further check on the computations.

Thus our procedure for computing the anomalous dimensions of the field operators to two-loop order is to evaluate the dimensionally regularized Feynman diagrams to this order, extract the coefficients of the single pole terms, and use Eq. (2.15). Note that for the N-loop contribution to $C_\Gamma^{(k)}$, it is true that

$$\sum_{\ell}\rho_\ell x_\ell\,\frac{\partial C_\Gamma^{(k)}}{\partial x_\ell}\bigg|_{N\text{-loop}} = 2N\,C_\Gamma^{(k)} \tag{2.17}$$

so that the derivatives need not be carried out explicitly.

The gauge dependence of the internal gauge field lines is incorporated in the gauge field propagator

$$D_{\mu\nu}(k) = \left(-g_{\mu\nu} + \alpha\,\frac{k_\mu k_\nu}{k^2}\right)\cdot\frac{1}{k^2} \tag{2.18}$$

with α a gauge parameter. The anomalous dimensions of the scalar and fermion fields are gauge-dependent, but this gauge-dependence will be cancelled in the evaluation of the Yukawa and scalar quartic coupling constants by the gauge-dependence of the proper vertex parts. The anomalous dimension of the background gauge field, which is equal to the β-function for the gauge-coupling, is gauge-independent when the modified Feynman rules for diagrams with external (background) gauge field lines are used (see [16]).

3. Scalar Wave Function Renormalization

The diagrams which contribute to the two-loop scalar wave function renormalization are shown in Figure 1 (g^4 terms) and Figure 2 (terms involving Yukawa and scalar quartic couplings). The contribution of a diagram to the singular part of the scalar wave function renormalization matrix can be expressed in the form

$$(\underset{\sim}{Z}_s^{-1})_{ab} = \frac{1}{(4\pi)^4} S_{ab}\left(\frac{A}{\eta^2} + \frac{B}{\eta}\right) \qquad (3.1)$$

where S_{ab} is a group theoretic factor associated with the diagram, and

$$\frac{1}{\eta} \equiv \frac{1}{\varepsilon} + \ln 4\pi - \gamma_E \qquad (3.2)$$

(γ_E = Euler constant) is the usual $\overline{MS}$ expansion parameter.

For diagram (1.1), the singular coefficients are given by

$$A\,S_{ab} = \frac{1}{2} g^4\, C_2(S)[(5+3/2\,\alpha)C_2(G) - 4\kappa\, S_2(F) - 1/2\, S_2(S)]\delta_{ab} \qquad (3.3)$$

and

$$B\,S_{ab} = -g^4 C_2(S)[(37/12 - 17/8\,\alpha + 3/8\,\alpha^2)C_2(G) - 5/3\,\kappa S_2(F) - 11/24\, S_2(S)]\delta_{ab}$$

where $C_2(S)$ is the Casimir operator of G acting on the scalar representation, $S_2(F)$ and $S_2(S)$ are the Dynkin indices for fermion and scalar representations, respectively (see Appendix A), and the factor $\kappa = 1/2$ for two-component fermions and $\kappa=1$ for four-component fermions. The coefficients of the singular terms from the remaining diagrams are given in Tables I and II (we note that the appropriate one-loop counterterms have been subtracted from each of the diagrams).

The two-loop anomalous dimension matrix for the scalar fields is extracted from these results by

$$\gamma^s_{ab}\Big|_{2\text{-loop}} = -\frac{2}{(4\pi)^4} \sum_{\text{diagrams}} B\,S_{ab} \qquad (3.5)$$

in view of Eqs. (2.15) and (2.17), with the change of sign from Eq. (2.15) due to the fact that we are here expanding $\underset{\sim}{Z}_s^{-1}$. There follows

$$\begin{aligned}(4\pi)^4\, \gamma^s_{ab}\Big|_{2\text{-loop}} = {} & \frac{1}{12}\lambda_{acde}\lambda_{bcde} \\ & -3\kappa\,\mathrm{Tr}[\underset{\sim}{Y}^b \underset{\sim}{Y}^{\dagger a} \underset{\sim}{Y}^c \underset{\sim}{Y}^{\dagger c}] - 2\kappa\,\mathrm{Tr}[\underset{\sim}{Y}^b \underset{\sim}{Y}^{\dagger c} \underset{\sim}{Y}^a \underset{\sim}{Y}^{\dagger c}] \\ & + 10\kappa g^2 \mathrm{Tr}[\underset{\sim}{C}_2(F)\underset{\sim}{Y}^a \underset{\sim}{Y}^{\dagger b}] \\ & - g^4\, C_2(S)\left\{\left(\frac{113}{12} + \frac{5}{2}\alpha - \frac{1}{4}\alpha^2\right) C_2(G)\right. \\ & \left. - \frac{10}{3}\kappa\, S_2(F) - \frac{11}{12} S_2(S) - \frac{3}{2}\; C_2(S)\right\}\delta_{ab} \end{aligned} \qquad (3.6)$$

with $\underset{\sim}{C}_2(f) \equiv \underset{\sim}{t}^A \underset{\sim}{t}^A$ the Casimir operator of G acting on the fermions. We note that γ^s_{ab} is in fact diagonal in the scalar fields (see Appendix A). The reader can verify that the double pole terms satisfy Eq. (2.16), if the gauge parameter α is included properly in the sum over coupling parameters. This two-loop result augments the well-known one-loop result

$$\begin{aligned}(4\pi)^2 \gamma^s_{ab}\Big|_{1\text{-loop}} = {} & 2\kappa\,\mathrm{Tr}(\underset{\sim}{Y}^a \underset{\sim}{Y}^{\dagger b}) \\ & - g^2(2+\alpha)C_2(S)\delta_{ab}. \end{aligned} \qquad (3.7)$$

4. Fermion Wave Function Renormalization

The diagrams which contribute to the two-loop fermion wave function renormalization are shown in Figure 3 (g^4, Y^4 terms) and Figure 4 (g^2Y^2 terms). The contribution of a diagram to the singular part of the fermion wave function renormalization matrix can be expressed as

$$Z_F^{-1} = \frac{1}{(4\pi)^4} S \left(\frac{A}{\eta^2} + \frac{B}{\eta}\right) \qquad (4.1)$$

where S is a group theoretic factor associated with the diagram, and η is the $\overline{MS}$ expansion parameter defined by Eq. (3.2).

Diagram (3.1) contributes only a single pole, with

$$B\,S = \frac{1}{4} g^4\, C_2(F)[(5 + 3/2\,\alpha) C_2(G) - 4\,\kappa S_2(F) - 1/2\, S_2(S)] \qquad (4.2)$$

where $C_2(F) = t^A t^A$ is the Casimir operator of G acting on the fermions. The double pole is absent for this diagram because the polarization tensor insertion projects out the transverse part of the vector propagator, and the one-loop divergence in the fermion wave function renormalization is absent in transverse (Landau) gauge.

The coefficients of the singular terms from the remaining diagrams are given in Tables III and IV (the appropriate one-loop counterterms have been subtracted from each of these diagrams).

The two-loop fermion anomalous dimension matrix is then given by

$$\gamma^F \Big|_{2\text{-loop}} = -\frac{2}{(4\pi)^4} \sum_{\text{diagrams}} B\, S \qquad (4.3)$$

as in Section 3 for the scalars. Using the results of Appendix A to rearrange the group theoretic factors, we obtain

$$\begin{aligned}(4\pi)^4\, \gamma^F \Big|_{2\text{-loop}} = & -\frac{1}{8} Y^a Y^{\dagger b} Y^b Y^{\dagger a} \\ & - \frac{3}{2}\kappa\, (\mathrm{Tr}\, Y^a Y^{\dagger b}) Y^{\dagger a} Y^b \\ & + g^2 [9/2\, C_2(S) Y^a Y^{\dagger a} - 7/4\, C_2(F) Y^a Y^{\dagger a} - 1/4\, Y^a C_2(F) Y^{\dagger a}] \\ & + g^4\, C_2(F) [1/2 (17 - 5\alpha + 1/2\,\alpha^2) C_2(G) - 2\,\kappa S_2(F) - 1/4\, S_2(S)] \\ & - \frac{3}{2} g^4 [C_2(F)]^2 \end{aligned} \qquad (4.4)$$

which is evidently diagonal in the fermion fields. Here $C_2(S) Y^a Y^{\dagger a}$ implies summation over scalar fields weighted by the appropriate eigenvalue of the Casimir operator.

This augments the one-loop result

$$(4\pi)^2 \gamma^F \Big|_{1\text{-loop}} = \frac{1}{2} Y^a Y^{\dagger a} + g^2\, C_2(F)(1-\alpha). \qquad (4.5)$$

It is amusing to note that for both fermion and scalar anomalous dimensions, the g^2Y^2 terms are actually independent of the gauge parameter α. It would be interesting to know if there is a general reason why this should be true.

5. Vector Wave Function Renormalization

The two-loop renormalization of gauge theories with fermions has been extensively studied, both in conventional gauges [8] and in background field gauge [16]. Indeed, even the three-loop β-function and anomalous dimensions have now been calculated for such theories [9].

We compute here the additional terms in the two-loop β-function in the presence of scalar fields. These terms are due to the diagrams of Figure 5, which involve the Yukawa couplings, and those of Figure 6, which involve only the gauge couplings of the scalar fields. We evaluate the diagrams of Figure 6 both in the usual R_ξ-gauge [18,19] and in the corresponding background field gauge (B_ξ-gauge) with the modified Feynman rules as given in [16]. The use of the B_ξ-gauge greatly simplifies the calculation of the β-function β_g for the gauge coupling, since only the divergent part of the wave function renormalization constant Z_A of the external background field, and the corresponding anomalous dimension

$$\gamma_A \equiv \frac{1}{2}\,\mu\,\frac{d}{d\mu}\,\ln Z_A \qquad (5.1)$$

need be computed explicitly. Then β_g is given simply by [16]

$$\beta_g = g\,\gamma_A \qquad (5.2)$$

The contribution of a diagram to the singular part of Z_A can be expressed as

$$Z_A^{-1} = \frac{1}{(4\pi)^4}\, S \left(\frac{A}{\eta^2} + \frac{B}{\eta}\right) \qquad (5.3)$$

where S is a group theoretic factor, and η is the $\overline{MS}$ expansion parameter defined by Eq. (3.2). The coefficients of the singular terms for the diagrams of Figure 5 are given in Table V; those for Figure 6 in Table VI. In Table VI, we give results in R_ξ-gauge together with the

additional terms due to the modified B_ξ-gauge Feynman rules. We also give the corresponding results for diagrams 6.1-6.4 with the scalar loop replaced by a fermion loop.

The two-loop anomalous dimension for the background field is then given by

$$\gamma_A\Big|_{2\text{-loop}} = \frac{2}{(4\pi)^4} \sum_{\text{diagrams}} BS. \qquad (5.4)$$

With the pure gauge theory contributions from [16], we then have

$$\begin{aligned}(4\pi)^4\gamma_A\Big|_{2\text{-loop}} &= -\frac{34}{3}\,g^4[C_2(G)]^2 \\ &+ \kappa g^4[4C_2(F) + 20/3\; C_2(G)]S_2(F) \\ &+ g^4[2\; C_2(S) + 1/3\; C_2(G)]S_2(S) \\ &- 2\,\kappa g^2\, Y_4(F) \end{aligned} \qquad (5.5)$$

where the invariant $Y_4(F)$ is defined by Equation (A.22) in Appendix A. The terms $C_2(F)S_2(F)$ and $C_2(S)S_2(S)$ imply summation over irreducible representations of fermions and scalars, respectively. The corresponding two-loop β-function for the gauge coupling is then obtained from Eq. (5.2).

The result (5.5) is independent of the gauge parameter as it must be. Moreover, the non-transverse parts of the polarization tensor which appear in the contributions from individual diagrams cancel in the final result. However, there are no gauge independent or purely transverse subsets of diagrams as there were in the calculation of the pure gauge theory diagrams by Abbott [16]. We note also that the double pole terms cancel in B_ξ gauge as they must; in R_ξ gauge, they satisfy Eq. (2.16).

6. Discussion and Conclusions

From the results of Section 5, the β-function for the coupling constant through two-loop order can now be written as

$$\beta(g) = \mu \frac{dg}{d\mu} =$$

$$= - \frac{g^3}{(4\pi)^2} \left\{ \frac{11}{3} C_2(G) - \frac{4}{3}\kappa\ S_2(F) - \frac{1}{6} S_2(S) + \frac{2\kappa}{(4\pi)^2} Y_4(F) \right\}$$

$$- \frac{g^5}{(4\pi)^4} \left\{ 34/3\ [C_2(G)]^2 - \kappa\ [4\ C_2(F) + 20/3\ C_2(G)] S_2(F) \right.$$

$$\left. - [2\ C_2(S) + 1/3\ C_2(G)] S_2(S) \right\}$$

$$+ \cdots \text{(higher-order terms)}. \qquad (6.1)$$

While it is difficult to estimate the size of the two-loop corrections in the absence of numerical solutions, we note that the scalars may contribute significantly.

Also, the qualitative effect of the Yukawa couplings is to push the gauge couplings toward asymptotic freedom, tending to offset the effects of fermions and scalars in the g^5 term. Thus if there exist heavy fermions or scalars (with mass $\sim m_W$), the question of two-loop corrections to the proton decay rate and quark-lepton mass ratios in grand unified gauge theories must be reexamined. (For a discussion of one-loop effects on m_b/m_τ, see [25] and references contained therein.)

For a gauge theory with fermions, our results for the β-function agree with the standard results [8], and our two-loop anomalous dimension for the fermions agrees with the result of Egoryan and Tarasov [11]

computed in a general gauge, and those of Tarrach and of Nachtmann and Wetzel in special gauges [11]. The contribution of scalar loops to the β-function agrees with that of Jones [26] if it is noted that our scalars are implicitly real, while his are complex. Comparisons with explicit calculations for $SU(3) \times SU(2) \times U(1)$ are given in Appendix B.

For a theory with a single Dirac fermion and a pseudoscalar field, with Yukawa and scalar couplings, our results for the anomalous dimensions agree with that of Vladimirov [13].

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Appendix A: Group Theoretic Results

Representation matrices $\underset{\sim}{T}^A$ for the generators of the gauge group G satisfy the commutation relations

$$[\underset{\sim}{T}^A, \underset{\sim}{T}^B] = i\, f^{ABC} \underset{\sim}{T}^C \qquad (A.1)$$

where the f^{ABC} are the structure constants of G. The (quadratic) Casimir operator

$$\underset{\sim}{C}_2 \equiv \underset{\sim}{T}^A \underset{\sim}{T}^A \qquad (A.2)$$

commutes with the generators of G, and is thus a multiple of the identity on each irreducible representation R of G, with eigenvalue denoted by $C_2(R)$. The Dynkin index $S_2(R)$ for an irreducible representation is defined by

$$\mathrm{Tr}_R\, \underset{\sim}{T}^A \underset{\sim}{T}^B = S_2(R)\delta^{AB} \qquad (A.3)$$

where the trace is taken only over R; evidently

$$d(G)S_2(R) = d(R)C_2(R) \qquad (A.4)$$

where d(R) is the dimension of R, and d(G) is the dimension of the Lie algebra of G.

For the computation of $S_2(R)$ and $C_2(R)$ for various groups and representations, see [27,28] and the references contained therein. In particular, note that the eigenvalue $C_2(G)$ of the Casimir operator on the regular (adjoint) representation of G is given by

$$f^{ACD} f^{BCD} = C_2(G)\delta^{AB} \qquad (A.5)$$

(this defines an overall normalization convention). Evidently

$$-i\, f^{ABC}\, \underset{\sim}{T}^A \underset{\sim}{T}^B \underset{\sim}{T}^C = \frac{1}{2} C_2(G)\underset{\sim}{C}_2 \qquad (A.6)$$

$$\underset{\sim}{T}^A \underset{\sim}{T}^B \underset{\sim}{T}^A \underset{\sim}{T}^B = [\underset{\sim}{C}_2 - 1/2\, C_2(G)]\underset{\sim}{C}_2 \qquad (A.7)$$

and, for an irreducible representation R,

$$-i\, f^{ACD}\, \mathrm{Tr}_R\, \underset{\sim}{T}^B \underset{\sim}{T}^C \underset{\sim}{T}^D = \frac{1}{2}\, C_2(G) S_2(R)\, \delta^{AB} \tag{A.8}$$

$$\mathrm{Tr}_R\, \underset{\sim}{T}^A \underset{\sim}{T}^C \underset{\sim}{T}^B \underset{\sim}{T}^C = [C_2(R) - 1/2\, C_2(G)]\, S_2(R) \delta^{AB}. \tag{A.9}$$

These relations are used to derive the group theoretic factors given in the tables. For scalar and fermion representations which are reducible (as will be the case in general), Eqs. (A.8) and (A.9) are to be replaced by a sum over irreducible representations.

If the gauge group is not simple, but is expressed as a direct product $G_1 \times \ldots \times G_n$, with corresponding gauge coupling constants $g_1, \ldots, g_n$, then it is necessary to make the substitutions

$$g^2 \underset{\sim}{C}_2 \to \sum_k g_k^2 \underset{\sim}{C}_2^k \tag{A.10}$$

$$g^4\, C_2(G) \underset{\sim}{C}_2 \to \sum_k g_k^4\, C_2(G_k) \underset{\sim}{C}_2^k \tag{A.11}$$

$$g^4\, S_2(R) \underset{\sim}{C}_2 \to \sum_k g_k^4\, S_2^k(R) \underset{\sim}{C}_2^k \tag{A.12}$$

$$g^4 [\underset{\sim}{C}_2]^2 \to \sum_{k,\ell} g_k^2 g_\ell^2\, \underset{\sim}{C}_2^k \underset{\sim}{C}_2^\ell \tag{A.13}$$

where $\underset{\sim}{C}_2^k$, S_2^k denote Casimir operator and index for the subgroup G_k. The substitution (A.12) is appropriate for the anomalous dimensions of the scalar and fermion fields; in the β-function for the gauge coupling constant g_ℓ, the corresponding substitution is

$$g^4\, C_2(R) S_2(R) \to \sum_k g_k^2 g_\ell^2\, C_2^k(R) S_2^\ell(R). \tag{A.14}$$

Evidently (A.13) - (A.14) lead to the interplay between the coupling constants of the different gauge groups in the two-loop RG equations.

The Yukawa coupling matrices $\underset{\sim}{Y}^a \equiv (Y_{jk}^a)$ must satisfy

$$Y_{jk}^b \theta_{ba}^A + Y_{j\ell}^a t_{\ell k}^A - t_{j\ell}^A Y_{\ell k}^a = 0 \tag{A.15}$$

in order to be gauge invariant; in matrix form,

$$[\underset{\sim}{t}^A, \underset{\sim}{Y}^a] = \underset{\sim}{Y}^b \theta_{ba}^A . \tag{A.16}$$

(the notation follows that of [29]). It then follows that

$$[\underset{\sim}{t}^A, \underset{\sim}{Y}^a \underset{\sim}{Y}^{\dagger a}] = 0 \tag{A.17}$$

whence

$$\underset{\sim}{Y}_2(F) \equiv \underset{\sim}{Y}^a \underset{\sim}{Y}^{\dagger a} \tag{A.18}$$

is a multiple of the identity on each irreducible fermion representation. Furthermore,

$$\theta_{ac}^A\, \mathrm{Tr}\, \underset{\sim}{Y}^c \underset{\sim}{Y}^{\dagger b} + \theta_{bc}^A\, \mathrm{Tr}\, \underset{\sim}{Y}^a \underset{\sim}{Y}^{\dagger c} = 0 \tag{A.19}$$

whence

$$\mathrm{Tr}\, \underset{\sim}{Y}^a \underset{\sim}{Y}^{\dagger b} \equiv Y_2(S) \delta^{ab} \tag{A.20}$$

is a multiple of the identity on each irreducible scalar representation.

It is then easy to see that

$$\begin{aligned} \underset{\sim}{t}^A \underset{\sim}{Y}^a \underset{\sim}{t}^A \underset{\sim}{Y}^{\dagger a} &= \underset{\sim}{Y}^a \underset{\sim}{t}^A \underset{\sim}{Y}^{\dagger a} \underset{\sim}{t}^A = \\ &= \frac{1}{2} [\underset{\sim}{C}_2(F) \underset{\sim}{Y}^a \underset{\sim}{Y}^{\dagger a} + \underset{\sim}{Y}^a\, \underset{\sim}{C}_2(F) \underset{\sim}{Y}^{\dagger a}] \\ &\quad - \frac{1}{2}\, C_2(S)\, \underset{\sim}{Y}^a \underset{\sim}{Y}^{\dagger a} \end{aligned} \tag{A.21}$$

where $C_2(S) \underset{\sim}{Y}^a \underset{\sim}{Y}^{\dagger a}$ implies summation over irreducible representations of the scalars, weighted by the corresponding eigenvalue of the Casimir operator. Equations (A.16) and (A.21) allow the reduction of the group theoretic factors in Table IV to a linear combination of the basic invariants

$$\underset{\sim}{C}_2(F)\underset{\sim}{Y}^a\underset{\sim}{Y}^{\dagger a},\ \underset{\sim}{Y}^a\underset{\sim}{C}_2(F)\underset{\sim}{Y}^{\dagger a},\ C_2(S)\underset{\sim}{Y}^a\underset{\sim}{Y}^{\dagger a}.$$

Also, introduce the invariants

$$Y_4(F) \equiv \frac{1}{d(G)}\,\mathrm{Tr}\ \underset{\sim}{C}_2(F)\underset{\sim}{Y}^a\underset{\sim}{Y}^{\dagger a} \qquad \text{(A.22)}$$

$$Y_4(S) \equiv \frac{1}{d(G)}\,\mathrm{Tr}\ C_2(S)\underset{\sim}{Y}^a\underset{\sim}{Y}^{\dagger a} \qquad \text{(A.23)}$$

[If the gauge group G is not simple, define invariants $Y_4^k(F)$ and $Y_4^k(S)$ for each simple factor group G_k]. The group theoretic factors which appear in Table V are then expressed as

$$\mathrm{Tr}\ \underset{\sim}{Y}^a\underset{\sim}{Y}^{\dagger a}\underset{\sim}{t}^A\underset{\sim}{t}^B = Y_4(F)\delta^{AB} \qquad \text{(A.24)}$$

$$\theta^A_{ab}\theta^B_{ca}\ \mathrm{Tr}\ \underset{\sim}{Y}^b\underset{\sim}{Y}^{\dagger c} = Y_4(S)\delta^{AB}. \qquad \text{(A.25)}$$

and

$$\mathrm{Tr}\ \underset{\sim}{Y}^a\underset{\sim}{t}^A\underset{\sim}{Y}^{\dagger a}\underset{\sim}{t}^B = [Y_4(F) - 1/2\ Y_4(S)]\delta^{AB} \qquad \text{(A.26)}$$

which follows directly by taking the trace of Eq. (A.21). Note that the invariant $Y_4(S)$ disappears from the β-function (6.1) for the gauge couplings.

Appendix B: Minimal SU(3) x SU(2) x U(1) Model

In the minimal version of the standard SU(3) x SU(2) x U(1) model, the left-handed fermions consist of quark and lepton doublets denoted by q and ℓ, respectively, and antiquark and antilepton singlets denoted by $\bar{u}$, $\bar{d}$, $\bar{e}$ in some number $n_g \geq 3$ of generations. The Yukawa couplings to the single scalar SU(2) doublet can be written as

$$y = -\bar{e}\ \underset{\sim}{F}_L\ \phi^\dagger \ell - \bar{d}\ \underset{\sim}{F}_D\ \phi^\dagger q$$
$$-\ \bar{u}\ \underset{\sim}{H}\ \phi^{c\dagger} q + \text{h.c.} \qquad \text{(B.1)}$$

where $\phi^c \equiv i\,\tau_2\phi^*$ is the conjugate scalar doublet, and $\underset{\sim}{F}_L$, $\underset{\sim}{F}_D$, $\underset{\sim}{H}$ are coupling matrices in generation space (the notation used is that of [25]).

The gauge couplings g_1, g_2, g_3 for the U(1), SU(2) and SU(3) subgroup then evolve according to equations of the form

$$\frac{dg_\ell}{dt} \equiv \beta_\ell(g) = -b_\ell\,\frac{g_\ell^3}{16\pi^2} - \sum_k b_{k\ell}\,\frac{g_k^2 g_\ell^3}{(16\pi^2)^2}$$
$$-\ \frac{g_\ell^3}{(16\pi^2)^2}\ \mathrm{Tr}[C_k^U \underset{\sim}{H}^\dagger \underset{\sim}{H} + C_k^D \underset{\sim}{F}_D^\dagger \underset{\sim}{F}_D + C_k^L\ \underset{\sim}{F}_L^\dagger \underset{\sim}{F}_L] \qquad \text{(B.2)}$$

where the last term represents the contribution of the Yukawa couplings. The trace is over fermion generations.

With conventional SU(5) normalization for the coupling constant $g_1 = \sqrt{3/5}\ g'_{W-S}$,

$$b_1 = -\frac{4}{3}n_g - \frac{1}{10} \qquad \text{(B.3a)}$$

$$b_2 = \frac{22}{3} - \frac{4}{3}n_g - \frac{1}{6} \qquad \text{(B.3b)}$$

$$b_3 = 11 - \frac{4}{3}n_g \qquad \text{(B.3c)}$$

Also, the coefficients $b_{k\ell}$ are given by the matrix

$$(b_{k\ell}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{136}{3} & 0 \\ 0 & 0 & 102 \end{pmatrix}$$

$$- n_g \begin{pmatrix} \frac{19}{15} & \frac{1}{5} & \frac{11}{30} \\ \frac{3}{5} & \frac{49}{3} & \frac{3}{2} \\ \frac{44}{15} & 4 & \frac{76}{3} \end{pmatrix} - \begin{pmatrix} \frac{9}{50} & \frac{3}{10} & 0 \\ \frac{9}{10} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (B.4)$$

where the last contribution is due to the scalar doublet. Finally, the coefficients C_k^F are given by the matrix

$$(C_k^F) = \begin{pmatrix} \frac{17}{10} & \frac{1}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{3}{2} & \frac{1}{2} \\ 2 & 2 & 0 \end{pmatrix} \qquad (B.5)$$

These results agree with those of Fischler and Hill [17] except that we have added the contribution of the scalar doublet to the matrix $(b_{k\ell})$, and we have corrected the contribution of the lepton Yukawa coupling to the evolution of the U(1) coupling constant. The anomalous dimensions of the fermions and scalars agree diagram by diagram with the calculations of Fischler and Oliensis [38].

Table I. Singular parts of the scalar wave function renormalization matrix $\underset{\sim}{Z}_S^{-1}$ defined by Eq. (3.1) from the diagrams in Figure 1 (with the appropriate one-loop counterterms subtracted). The group theoretic factor $S_{ab} = S\,\delta_{ab}$, with S given the table.

Diagram	S	A	B
1.1		Eq. (3.3)	Eq. (3.4)
1.2	$g^4[C_2(S)]^2$	$\frac{1}{2}(2+\alpha)^2$	$-\frac{1}{4}(2+\alpha)^2$
1.3	$g^4\, C_2(S)[C_2(S) - \frac{1}{2}C_2(G)]$	$(2+\alpha)(1-\alpha)$	$\frac{1}{2} - 3\alpha + \frac{7}{4}\alpha^2$
1.4	$\frac{1}{2}g^4\, C_2(S)C_2(G)$	$\frac{3}{2}(2+\alpha)(1-\alpha)$	$-\frac{17}{8} - \frac{31}{4}\alpha + 2\alpha^2$
1.5a +1.5b +1.6	$g^4\, C_2(S)[2C_2(S) - \frac{1}{2}C_2(G)]$	$-\frac{3}{2}(2+\alpha)$	$-\frac{3}{8} + \frac{5}{2}\alpha - \alpha^2$
1.7	$g^4\, C_2(S)[2C_2(S) - \frac{1}{2}C_2(G)]$	0	$1 - \frac{1}{2}\alpha + \frac{1}{4}\alpha^2$

Table II. Singular parts of the scalar wave function renormalization matrix $\underset{\sim}{Z}_S^{-1}$ defined by Eq. (3.1) from the diagrams in Figure 2 (with appropriate one-loop counterterms subtracted). Here $\kappa = \frac{1}{2}$ (1) for two (four)-component fermions.

Diagram	S_{ab}	A	B
2.1	$\frac{1}{6}\lambda_{acde}\lambda_{bcde}$	0	$\frac{1}{4}$
2.2	$\kappa\, \mathrm{Tr} Y^b Y^{\dagger a} Y^c Y^{\dagger c}$	1	$-\frac{3}{2}$
2.3	$\kappa\, \mathrm{Tr} Y^b Y^{\dagger c} Y^a Y^{\dagger c}$	2	-1
2.4	$\kappa\, g^2 C_2(S)\, \mathrm{Tr} Y^a Y^{\dagger b}$	$-(2+\alpha)$	$1+\frac{5}{2}\alpha$
2.5	$\kappa\, g^2\, \mathrm{Tr}\, C_2(F) Y^a Y^{\dagger b}$	$2(1-\alpha)$	$-1+\alpha$
2.6	$\kappa\, g^2 [\mathrm{Tr}\, C_2(F) Y^a Y^{\dagger b} - \frac{1}{2} C_2(S)\, \mathrm{Tr} Y^a Y^{\dagger b}]$	$-2(4-\alpha)$	$2(3-1/2\alpha)$
2.7a +2.7b	$\kappa\, g^2 C_2(S)\, \mathrm{Tr} Y^a Y^{\dagger b}$	$2(1+2\alpha)$	$2(1-3/2\,\alpha)$

Table III. Singular parts of the fermion wave function renormalization matrix $\underset{\sim}{Z}_F^{-1}$ defined by Eq. (4.1) from the diagrams in Figure 3.

Diagram	S	A	B
3.1		0	Eq. (4.2)
3.2	$g^4 [C_2(F)]^2$	$\frac{1}{2}(1-\alpha)^2$	$-\frac{1}{4}(1-\alpha)^2$
3.3	$g^4\, C_2(F)[C_2(F) - \frac{1}{2} C_2(G)]$	$-(1-\alpha)^2$	$-\frac{1}{2}(1+\alpha-\frac{1}{2}\alpha^2)$
3.4	$\frac{1}{2} g^4 C_2(F) C_2(G)$	$-\frac{3}{2}(2-\alpha)(1-\alpha)$	$\frac{11}{2}-\frac{15}{4}\alpha+\frac{1}{2}\alpha^2$
3.5	$Y^a Y^{\dagger b} Y^b Y^{\dagger a}$	$\frac{1}{8}$	$-\frac{1}{16}$
3.6	$\kappa\, Y^a Y^{\dagger b}\, \mathrm{Tr} Y^{\dagger a} Y^b$	$\frac{1}{2}$	$-\frac{3}{4}$
3.7	$Y^a Y^{\dagger b} Y^a Y^{\dagger b}$	$\frac{1}{2}$	0

Table IV. Singular parts of the fermion wave function renormalization matrix $\underset{\sim}{Z}_F^{-1}$ defined by Eq. (4.1) from the diagrams in Figure 4.

Diagram	$\underset{\sim}{S}$	A	B
4.1	$g^2\, \underset{\sim}{C}_2(F) Y^a Y^{\dagger a}$	$\frac{1}{4}(1-\alpha)$	$-\frac{3}{8}(1-\alpha)$
4.2	$g^2\, \underset{\sim}{Y}^a \underset{\sim}{C}_2(F) \underset{\sim}{Y}^{\dagger a}$	$\frac{1}{4}(1-\alpha)$	$\frac{1}{8}(1-\alpha)$
4.3	$g^2\, \underset{\sim}{Y}^b \underset{\sim}{Y}^{\dagger a} \theta^A_{bc} \theta^A_{ca}$	$-\frac{1}{4}(2+\alpha)$	$\frac{1}{4}(3-\frac{1}{2}\alpha)$
4.4	$g^2\, \underset{\sim}{Y}^b \underset{\sim}{t}^A \underset{\sim}{Y}^{\dagger a}\, \theta^A_{ba}$	$\frac{1}{2}(1-\alpha)$	$-\frac{1}{4}(5+\alpha)$
4.5a +4.5b	$g^2(\underset{\sim}{Y}^b \underset{\sim}{Y}^{\dagger a} \underset{\sim}{t}^A\, \theta^A_{ba}$ $+ \underset{\sim}{t}^A \underset{\sim}{Y}^b \underset{\sim}{Y}^{\dagger a}\, \theta^A_{ba})$	$-\frac{1}{2}(1-\alpha)$	$\frac{1}{8}(4-\alpha)$
4.6a +4.6b	$g^2(\underset{\sim}{t}^A \underset{\sim}{Y}^a \underset{\sim}{t}^A \underset{\sim}{Y}^{\dagger a}$ $+ \underset{\sim}{Y}^a \underset{\sim}{t}^A \underset{\sim}{Y}^{\dagger a} \underset{\sim}{t}^A)$	$-\frac{1}{4}(5-2\alpha)$	$-\frac{1}{8}(3+\alpha)$

Table V: Singular parts of the vector wave function renormalization constant Z_A^{-1} defined by Eq. (5.3) from the diagrams in Figure 5.

Diagram	$S_{AB} \equiv S\, \delta_{AB}$	A	B
5.1	$\kappa\, g^2\, \mathrm{Tr}\, \underset{\sim}{Y}^a \underset{\sim}{Y}^{\dagger a} \underset{\sim}{t}^A \underset{\sim}{t}^B$	$\frac{2}{3}$	$-\frac{10}{9}$
5.2	$\kappa\, g^2\, \theta^A_{ab} \theta^B_{ca}\, \mathrm{Tr}\, \underset{\sim}{Y}^b \underset{\sim}{Y}^{\dagger c}$	$\frac{1}{3}$	$-\frac{7}{18}$
5.3	$\kappa\, g^2\, \mathrm{Tr}\, \underset{\sim}{Y}^a \underset{\sim}{t}^A \underset{\sim}{Y}^{\dagger a} \underset{\sim}{t}^B$	$-\frac{2}{3}$	$\frac{1}{9}$
5.4a +5.4b	$\kappa\, g^2\, \theta^A_{ba}\, \mathrm{Tr}\, \underset{\sim}{Y}^b \underset{\sim}{Y}^{\dagger a} \underset{\sim}{t}^A$	$-\frac{4}{3}$	$\frac{8}{9}$

Table VI. Singular parts of the vector wave function renormalization constant Z_A^{-1} defined by Eq. (5.3) from the diagrams in Figure 6 and diagrams 6.1F–6.4F obtained from 6.1–6.4 by changing the scalar loop into a fermion loop. The additional contributions to diagrams 6.2-3, 6.2F–6.3F from the extra vertex terms in B_ξ gauge are noted explicitly.

Diagram	$S_{AB} \equiv S\delta_{AB}$	A	B
6.1	$g^4\, C_2(S)S_2(S)$	$-\frac{1}{6}(2+\alpha)$	$\frac{1}{9}(4-\alpha)$
6.2	$g^4\, C_2(G)S_2(S)$	$-\frac{7}{24}-\frac{1}{24}\alpha$	$\frac{55}{144}+\frac{31}{144}\alpha$
	(B_ξ)	$-\frac{1}{8}$	$+\frac{5}{16}$
6.3a +6.3b	$g^4\, C_2(G)S_2(S)$	$\frac{1}{8}+\frac{5}{24}\alpha$	$-\frac{5}{48}-\frac{29}{144}\alpha$
	(B_ξ)	$+\frac{1}{3}-\frac{1}{12}\alpha$	$-\frac{1}{12}$
6.4	$g^4[C_2(S)-\frac{1}{2}C_2(G)]S_2(S)$	$-\frac{1}{6}(1-\alpha)$	$\frac{29}{36}-\frac{1}{18}\alpha$
6.5	$g^4[2C_2(S)-\frac{1}{2}C_2(G)]S_2(S)$	0	$\frac{1}{3}-\frac{1}{12}\alpha$
6.6a +6.6b	$g^4[2C_2(S)-\frac{1}{2}C_2(G)]S_2(S)$	$\frac{1}{4}$	$-\frac{11}{24}+\frac{1}{6}\alpha$
6.1F	$\kappa\, g^4 C_2(F)S_2(F)$	$\frac{4}{3}(1-\alpha)$	$-\frac{10}{9}(1-\alpha)$
6.2F	$\kappa\, g^4 C_2(G)S_2(F)$	$-\frac{7}{3}-\frac{1}{3}\alpha$	$\frac{13}{18}+\frac{25}{18}\alpha$
	(B_ξ)	-1	$+\frac{3}{2}$
6.3F	$\kappa\, g^4 C_2(G)S_2(F)$	$\frac{5}{3}\alpha$	$\frac{10}{3}-\frac{35}{18}\alpha$
	(B_ξ)	$+\frac{2}{3}(4-\alpha)$	$-\frac{2}{3}$
6.4F	$\kappa\, g^4[C_2(F)-\frac{1}{2}C_2(G)]S_2(F)$	$-\frac{4}{3}(1-\alpha)$	$\frac{28}{9}-\frac{10}{9}\alpha$

Figure Captions

Figure 1: Scalar wave function renormalization diagrams of order g^4.

Figure 2: Scalar wave function renormalization diagrams of order λ^2, Y^4, g^2Y^2.

Figure 3: Fermion wave function renormalization diagrams of order g^4, Y^4.

Figure 4: Fermion wave function renormalization diagrams of order g^2Y^2.

Figure 5: Gauge field wave function renormalization diagrams of order g^2Y^2.

Figure 6: Gauge field wave function renormalization diagrams of order g^4.

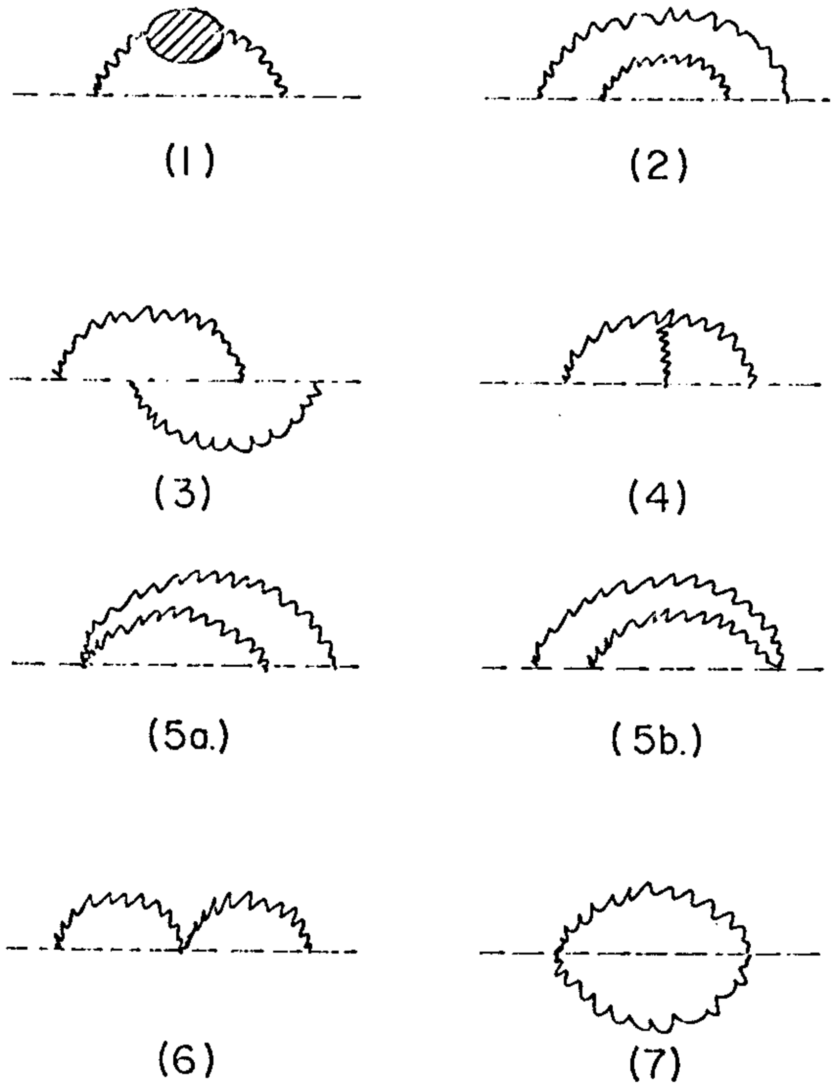

Fig. 1

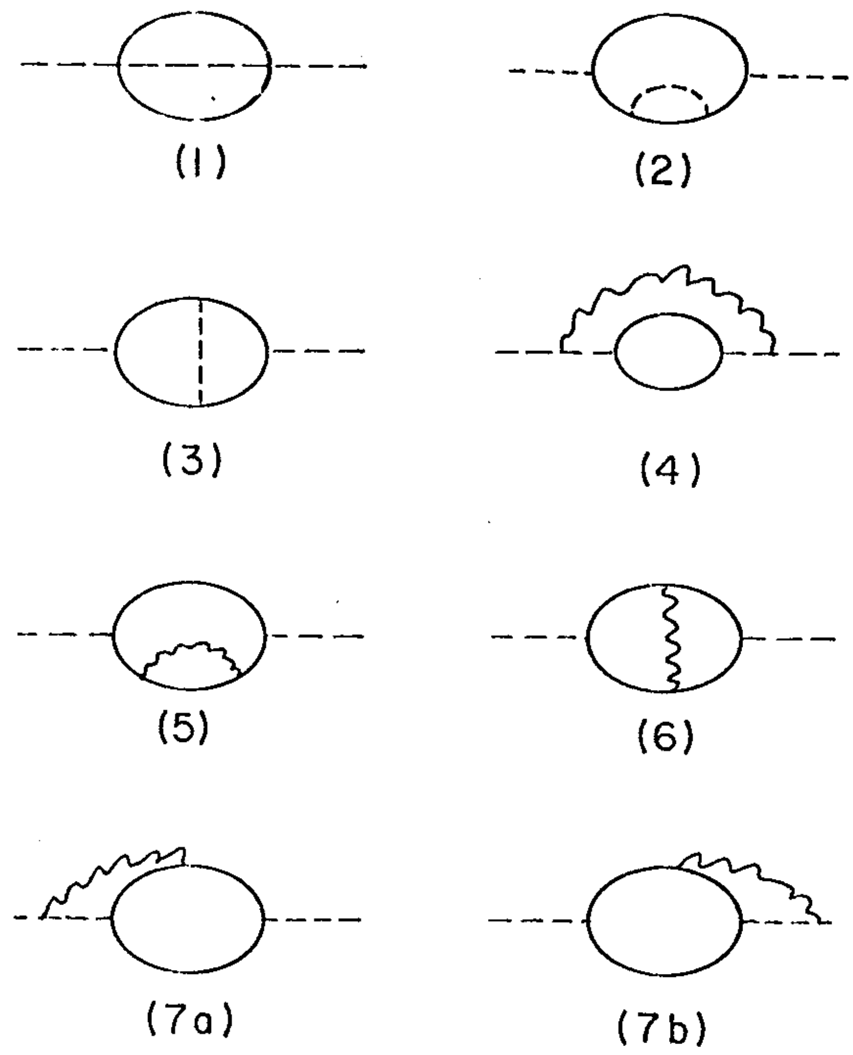

Fig. 2

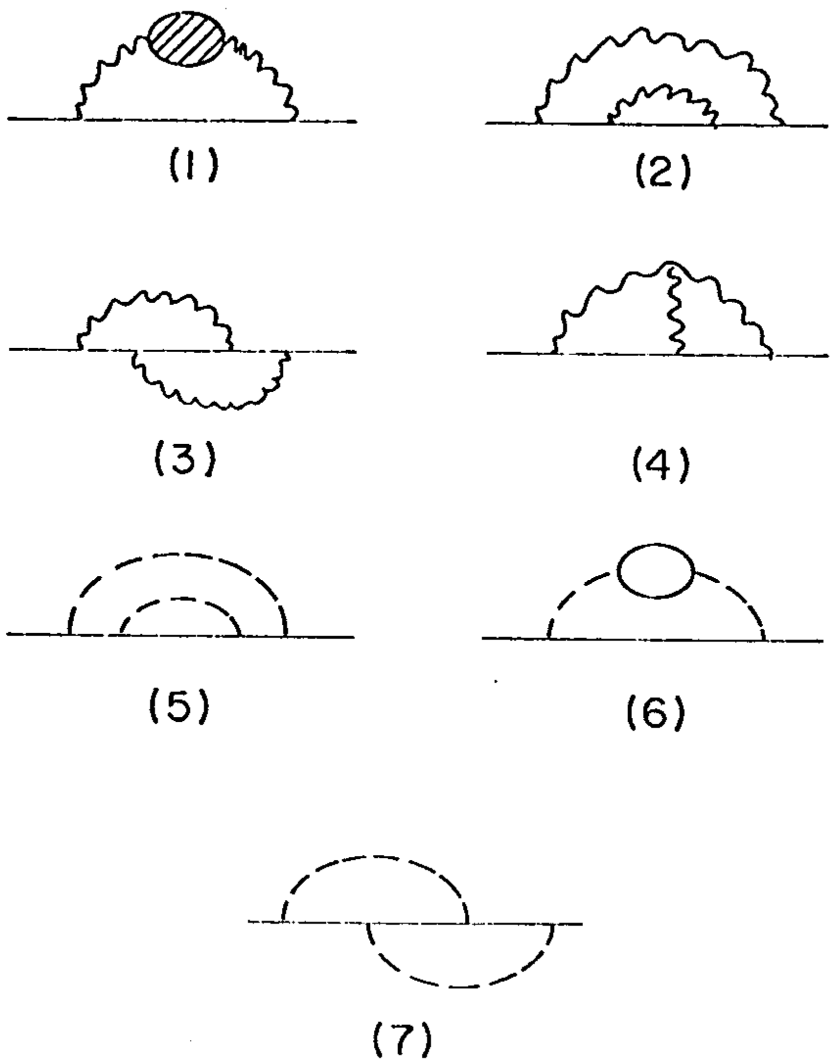

Fig. 3

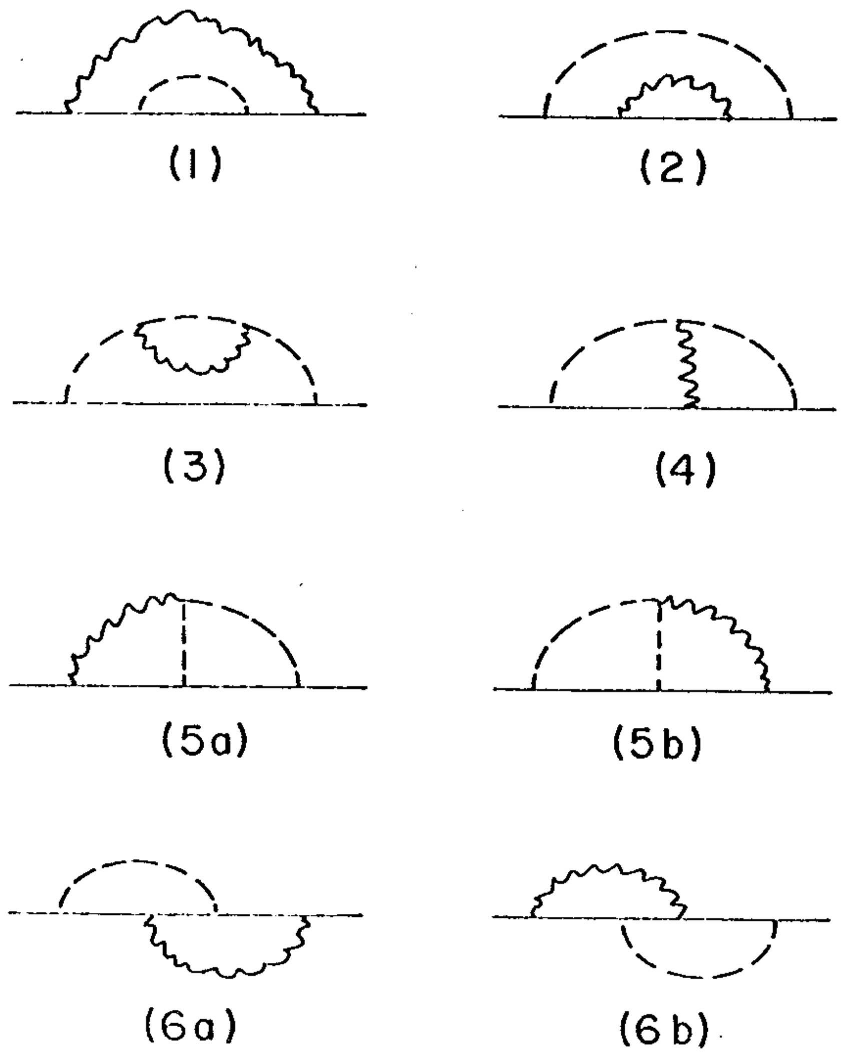

Fig. 4

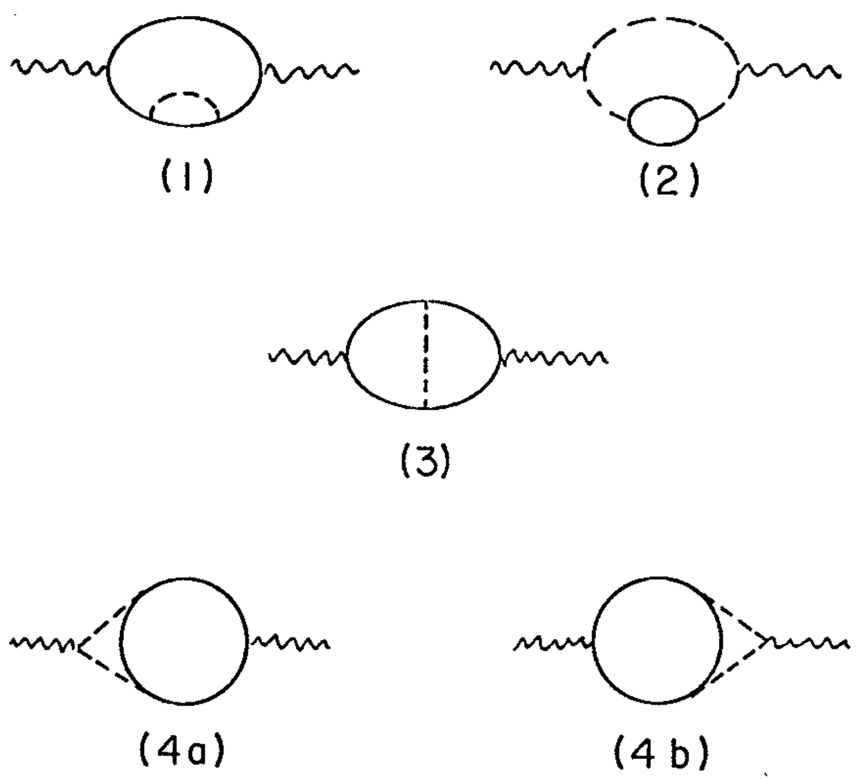

Fig. 5

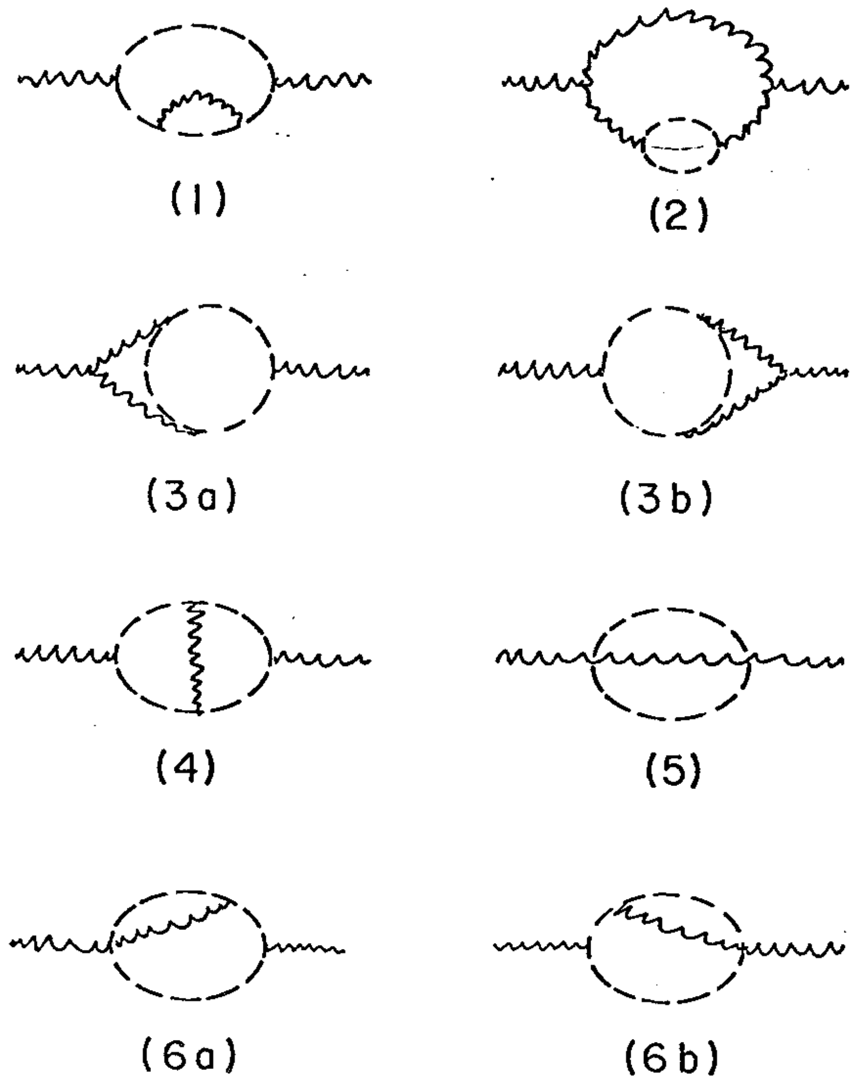

Fig. 6