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TWO-LOOP RENORMALIZATION GROUP EQUATIONS IN A GENERAL QUANTUM FIELD THEORY II. YUKAWA COUPLINGS

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ABSTRACT

The two-loop β-functions for the Yukawa couplings are computed in a general renormalizable quantum field theory with scalar, spin 1/2, and (vector) gauge fields associated with a general gauge group G. A more explicit form is given for the two-loop β-functions for the Yukawa couplings of the Higgs doublet in the minimal QCD-electroweak theory based on SU(3) x SU(2) x U(1).

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1. Introduction

In a recent paper [1], hereinafter referred to as I, we began an analysis of the two-loop evolution equations for the coupling constants, and anomalous dimensions of the field operators, in a general renormalizable quantum field theory with scalars, spin 1/2 fermions, and (vector) gauge fields associated with an arbitrary semi-simple gauge group G. In I, we computed the anomalous dimensions of the fields in a general R_ξ gauge, and of the gauge field in a background field gauge, to two-loop order, consequently obtaining the two-loop β-function for the gauge coupling including the contributions of the Yukawa couplings.

In the present work, we present a derivation of the two-loop β-functions for the Yukawa couplings of the scalar fields to the spin 1/2 fields, using dimensional regularization [2] and the modified minimal subtraction ($\overline{MS}$) scheme [3] for extracting the divergent parts of the vertex diagrams (see I for the relation between these divergent parts and the β-functions). The corresponding two-loop β-functions for the scalar quartic couplings will be given in a subsequent paper.

In Section 2, we recall our notation from I, and compute the singular parts of the two-loop proper vertex diagrams. These, together with the anomalous dimensions of the field operators computed in I, lead to the evaluation of the β-functions given in Section 3. In Section 4, we discuss our results, and compare them with partial results in the existing literature. In Appendix A, we collect some useful group theoretic results. In Appendix B, we give explicit β-functions for the Yukawa couplings of the Higgs doublet in the minimal QCD- electroweak theory based on the gauge group SU(3) x SU(2) x U(1).

2. Renormalization of the Proper Yukawa Vertex

As in I, we consider a general renormalizable field theory with vector gauge fields V_μ^A associated with a compact simple or semi-simple gauge group G, scalar fields ϕ_a, and (two-component) spinor fields ψ_j. The representations (reducible, in general) of G under which the scalar and spinor fields transform will be denoted by S and F, respectively.

The gauge-invariant Lagrange density of the theory has the general form

$$\mathcal{L} = -\frac{1}{4}F^A_{\mu\nu}F^A_{\mu\nu} + \frac{1}{2}D_\mu\phi_a D_\mu\phi_a + i\psi_j^\dagger\sigma_\mu D_\mu\psi_j$$

$$-(Y^a_{jk}\psi_j\zeta\psi_k\phi_a + \text{H.C.}) - \frac{1}{4!}\lambda_{abcd}\phi_a\phi_b\phi_c\phi_d \tag{2.1}$$

$$+\ .\ .\ .\ (\text{mass terms})$$

$$+\ .\ .\ .\ (\text{gauge-fixing + ghost terms})$$

where the mass terms are not relevant to the present calculation, and the gauge-fixing and ghost terms are here those appropriate to a standard R_ξ gauge [4]. Here

$$F^A_{\mu\nu} = \partial_\mu V_\nu^A - \partial_\nu V^A_\mu + gf^{ABC}V^B_\mu V^C_\nu \tag{2.2}$$

where the f^{ABC} are the structure constants of the gauge group, and g is the gauge coupling constant (the modifications of our results if the gauge group

is only semi-simple are explained below). Also,

$$D_\mu\phi_a = \partial_\mu\phi_a + ig\,\theta^A_{ab}V^A_\mu\phi_b \tag{2.3}$$

$$D_\mu\psi_j = \partial_\mu\psi_j + ig\,t^A_{jk}V^A_\mu\psi_k \tag{2.4}$$

where $\theta^A \equiv (\theta^A_{ab})$ and $t^A \equiv (t^A_{jk})$ are the (Hermitian) generators of the gauge group acting on the scalar and fermion fields, respectively. We implicitly assume a real representation for the scalars, so that the θ^A are imaginary and antisymmetric. The Yukawa coupling matrices

$$Y^a \equiv (Y^a_{jk}) \tag{2.5}$$

satisfy the basic invariance relation

$$[t^A, Y^a] = Y^b\theta^A_{ba} \tag{2.6}$$

which insures that the Yukawa coupling is invariant under the gauge group G (some useful consequences of this relation are derived in Appendix A). Finally, $\zeta = \pm i\sigma_2$ is the spinor metric.

The gauge-dependence of the internal gauge field lines is incorporated in the gauge field propagator

$$D_{\mu\nu}(k) = \left(-g_{\mu\nu} + \alpha\frac{k_\mu k_\nu}{k^2}\right)\cdot\frac{1}{k^2} \tag{2.7}$$

with α a gauge parameter. The proper Yukawa vertex will be gauge-dependent, but this gauge-dependence will be cancelled in the renormalization of the Yukawa coupling constants by the gauge-dependence of the anomalous dimensions of the scalar and fermion fields as computed in I.

The one-loop diagrams which modify the proper Yukawa vertex are shown in Figure 1. The corresponding two-loop diagrams can be divided into three classes:

(i) (one-loop) propagator insertions in the one-loop diagrams of Figure 1,

(ii) (one-loop) vertex insertions in the diagrams of Figure 1, and

(iii) diagrams with crossed ladder topology shown in Figure 2.

Diagrams of classes (i) and (ii) will be denoted as (m/Xn), where m is the one-loop diagram in which an insertion is made, X = P(V) for a propagator (vertex) insertion, and n = 1, 2, or 3 according to which line or vertex is modified by the insertion, as indicated in Figure 3.

The contributions of the diagrams to the renormalization of the proper Yukawa vertex are computed using dimensional regularization [2] and a mass-independent renormalization scheme [5] based on the modified minimal subtraction ($\overline{MS}$) algorithm [3]. The singular part of a diagram (evaluated in $d \equiv 4-2\varepsilon$ dimensions) can be expressed in the form

$$\underset{\sim}{Y}_2^a = \frac{1}{(4\pi)^4}\, \underset{\sim}{S}^a \left\{\frac{A}{\eta^2} + \frac{B}{\eta}\right\}, \tag{2.8}$$

where $\underset{\sim}{S}^a$ is a group-theoretic factor associated with the diagram, and

$$\frac{1}{\eta} \equiv \frac{1}{\varepsilon} + \ell n 4\pi - \gamma_E \tag{2.9}$$

(γ_E = Euler constant) is the usual $\overline{MS}$ expansion parameter. For the insertion diagrams, these singular parts are in fact a sum of terms of this form with different group-theoretic factors.

The singular parts of the propagator insertion diagrams are given in Table I, and those of the vertex insertion diagrams in Table II. (Note that the one-loop insertions include the appropriate $\overline{MS}$ counterterms.) For the diagram (2/P1), which is obtained by inserting the one-loop correction to the gauge field propagator in diagram (2) of Figure 1, the singular coefficients are given by

$$A = -\frac{1}{4}[(10 + 3\alpha)C_2(G) - 8\kappa S_2(F) - S_2(S)] \tag{2.10}$$

$$B = \left\{\frac{13}{3} - \frac{7}{4}\alpha + \frac{3}{8}\alpha^2\right\}C_2(G) - \frac{8}{3}\kappa S_2(F) - \frac{7}{12}S_2(S) \tag{2.11}$$

Here $C_2(G)$ is the Casimir operator of G in the adjoint representation, and $S_2(F)$, $S_2(S)$ are the Dynkin indices for the fermion and scalar representations, respectively, defined by

$$\mathrm{Tr}(\underset{\sim}{t}^A \underset{\sim}{t}^B) = S_2(F)\delta^{AB} \tag{2.12}$$

$$\mathrm{Tr}(\underset{\sim}{\theta}^A \underset{\sim}{\theta}^B) = S_2(S)\delta^{AB} \tag{2.13}$$

κ is a factor included in all diagrams which contain a fermion loop; $\kappa = 1/2$ or 1 for two-component or four-component fermions, respectively. Also appearing in the tables are the Casimir operators

$$C_2(F) \equiv t^A t^A \quad (2.14)$$

$$C_2^{ab}(S) \equiv \theta^A_{ac}\theta^A_{cb} = C_2(S)\delta^{ab} \quad (2.15)$$

for the fermions and scalars, as well as the quadratic invariants

$$Y_2(F) \equiv Y^{\dagger a}Y^a \quad (2.16)$$

$$Y_2^{ab}(S) \equiv \mathrm{Tr}\,Y^{\dagger a}Y^b = Y_2(S)\delta^{ab} \quad (2.17)$$

constructed from the Yukawa coupling matrices.

The singular parts of the crossed ladder diagrams are given in Table III. Note that these diagrams have at most a single pole in ϵ, since the integration over the first loop momentum is convergent for any routing of the loop momenta.

In these results, an implicit factor of the gauge coupling constant g is to be associated with each generator t^A or θ^A, and a corresponding factor g^2 with each Casimir operator or Dynkin index. If the gauge group is not simple, but a direct product $G_1 \times \ldots \times G_n$ of simple groups with corresponding gauge coupling constants $g_1,\ldots,g_n$, then it is necessary to make the substitutions

$$g^2C_2(R) \to \sum_k g_k^2 C_2^k(R) \quad (2.18)$$

$$g^4C_2(G)C_2(R) \to \sum_k g_k^4 C_2(G_k)C_2^k(R) \quad (2.19)$$

$$g^4C_2(R)S_2(R) \to \sum_k g_k^4 C_2^k(R)S_2^k(R) \quad (2.20)$$

$$g^4C_2(R)C_2(R') \to \sum_{k,\ell} g_k^2 g_\ell^2 C^k(R)C_2^{\ell}(R') \quad (2.21)$$

where C_2^k, S_2^k denote Casimir operator and index for the subgroup G_k (R,R' may be scalar or fermion representations).

3. Evaluation of the β-functions.

From the single-pole parts of the proper vertex corrections we can read off directly the anomalous dimensions $\gamma_Y^a \equiv (\gamma_Y{}^a{}_{jk})$ of the operators $\bar\psi_j\psi_k\phi_a$ as

$$\gamma_Y^a\Big|_{2\text{-loop}} = -\frac{4}{(4\pi)^4}\sum_{\text{diagrams}} BS^a \quad (3.1)$$

as explained in I. The corresponding β-functions for the Yukawa coupling matrices include contributions from the anomalous dimensions of the field operators, so that

$$\beta^a = \frac{dY^a}{dt} = \gamma_Y^a + \gamma^{\dagger F}Y^a + Y^a\gamma^F + \gamma^S_{ab}Y^b \quad (3.2)$$

and the γ^F, γ^S_{ab} have been given to two-loop order in I. The β^a are required to be independent of the gauge parameter α, and the cancellation of the gauge-dependence of γY^a with the gauge-dependence of the anomalous dimensions of the field operators is a useful check.

The two-loop β-function which results after combining terms is then given by

$$(4\pi)^4\beta^a\Big|_{2\text{-loop}} = 2Y^cY^{\dagger b}Y^a(Y^{\dagger c}Y^b - Y^{\dagger b}Y^c) \tag{3.3}$$

$$- Y^b[Y_2(F)Y^{\dagger a} + Y^{\dagger a}Y_2^{\dagger}(F)]Y^b$$

$$- \frac{1}{8}[Y^bY_2(F)Y^{\dagger b}Y^a + Y^aY^{\dagger b}Y_2^{\dagger}(F)Y^b]$$

$$- 4\kappa Y_2^{ac}(S)Y^bY^{\dagger c}Y^b - \frac{3}{2}\kappa Y_2^{bc}(S)[Y^bY^{\dagger c}Y^a + Y^aY^{\dagger c}Y^b]$$

$$- \kappa Y^b\mathrm{Tr}\{\frac{3}{2}[Y_2(F)Y^{\dagger b} + Y^{\dagger b}Y_2^{\dagger}(F)]Y^a + 2Y^{\dagger b}Y^cY^{\dagger a}Y^c\}$$

$$- 2\lambda_{abcd}Y^bY^{\dagger c}Y^d + \frac{1}{12}\lambda_{acde}\lambda_{bcde}Y^b$$

$$+ 3g^2\{C_2(F), Y^bY^{\dagger a}Y^b\} + 5g^2Y^b\{C_2(F), Y^{\dagger a}\}Y^b$$

$$- \frac{7}{4}g^2[C_2(F)Y_2^{\dagger}(F)Y^a + Y^aY_2(F)C_2(F)]$$

$$- \frac{1}{4}g^2[Y^bC_2(F)Y^{\dagger b}Y^a + Y^aY^{\dagger b}C_2(F)Y^b]$$

$$+ 6g^2[t^AY^aY^{\dagger b}t^AY^b + Y^bt^AY^{\dagger b}Y^at^A]$$

$$+ 5\kappa g^2Y^b\mathrm{Tr}\{C_2(F), Y^a\}Y^{\dagger b}$$

$$+ 6g^2[C_2^{bc}(S)Y^bY^{\dagger a}Y^c - 2C_2^{ac}(S)Y^bY^{\dagger c}Y^b]$$

$$+ \frac{9}{2}g^2[Y^bY^{\dagger c}Y^a + Y^aY^{\dagger c}Y^b]$$

$$- \frac{3}{2}g^4\{[C_2(F)]^2, Y^a\}$$

$$+ g^4[6C_2(S) - \frac{97}{6}C_2(G) + \frac{10}{3}\kappa S_2(F) + \frac{11}{12}S_2(S)]\{C_2(F), Y^a\}$$

$$- g^4C_2(S)[\frac{21}{2}C_2(S) - \frac{49}{4}C_2(G) + 2\kappa S_2(F) + \frac{1}{4}S_2(S)]Y^a$$

where the various quantities appearing in this equation have been defined in Section 2. Note also that the modifications required if the gauge group is not simple are given by Equations (2.18) - (2.21).

This augments the well-known one-loop result [6,7]

$$(4\pi)^2 \underset{\sim}{\beta}^a \Big|_{1\text{-loop}} = \tfrac{1}{2}[\underset{\sim}{Y}_2^\dagger(F)\underset{\sim}{Y}^a + \underset{\sim}{Y}^a\underset{\sim}{Y}_2(F)] + 2\underset{\sim}{Y}^b\underset{\sim}{Y}^{\dagger a}\underset{\sim}{Y}^b + 2\kappa\underset{\sim}{Y}^b \mathrm{Tr}\underset{\sim}{Y}^{\dagger b}\underset{\sim}{Y}^a - 3g^2\{\underset{\sim}{C}_2(F),\underset{\sim}{Y}^a\}. \tag{3.4}$$

For a more explicit evaluation of the β-function in the standard SU(3) x SU(2) x U(1) QCD-electroweak theory, see Appendix B.

4. Discussion and Conclusions

The central result of our paper is Equation (3.3), which gives the two-loop β-function for the Yukawa couplings in a general renormalizable field theory in terms of the Yukawa coupling matrices $\underset{\sim}{Y}^a$ of the theory, the scalar quartic couplings λ, the gauge coupling constants g, and group theoretic invariants associated with the representations of the fermion and scalar fields.

In order to facilitate the use of the Yukawa coupling matrices, we note that in a formulation based on four-component Dirac fermions, we can write simply

$$\underset{\sim}{Y}^a = \underset{\sim}{S}^a + i\underset{\sim}{P}^a\gamma_5 \tag{4.1}$$

$$\underset{\sim}{Y}^{\dagger a} = \underset{\sim}{S}^{\dagger a} - i\underset{\sim}{P}^{\dagger a}\gamma_5 \tag{4.2}$$

and treat the matrix element $Y^a{}_{jk}$ as the amplitude for annihilating a fermion of type k and creating a fermion of type j.

In the two-component formulation of Equation (2.1), $Y^a{}_{jk}$ is the amplitude for annihilating left-handed fermions of types j,k (and a scalar of type a). In order to follow a fermion line continuously, it is also useful to consider $Y^a{}_{jk}$ as the amplitude for annihilating a left-handed fermion of type k and creating a right-handed fermion of type j. Then also $(\underset{\sim}{Y}^{\dagger a})_{\ell m}$ is the amplitude for annihilating a right-handed fermion of type m and creating a left-handed fermion of type ℓ.

It is then clear that along a single fermion line (with alternating left- and right-handed fermions in the two-component formulation), the Yukawa coupling matrices must appear alternately as $\underset{\sim}{Y}$ and $\underset{\sim}{Y}^\dagger$, starting with $\underset{\sim}{Y}$ on an incoming left-handed fermion line. Here we have been careful to follow this rule consistently, although we were not so precise in I.

For complex representations of fermions or scalars, it is also useful to distinguish between upper and lower indices (although we have suppressed the distinction here for notational simplicity), since only contractions of an upper index with a lower index are allowed. Thus certain diagrams will actually vanish in theories with complex scalars, since the implied contractions in the group factors involve a pair of upper (or a pair of lower) indices.

The results obtained here agree with those obtained by Vladimirov [8] for a theory with a single Dirac fermion and single scalar field (no gauge fields). Our results also agree diagram by diagram with a recent calculation by Fischler and Oliensis [9], which is, however, explicitly restricted to SU(3) x SU(2) x U(1).

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APPENDIX A: Group Theoretic Results

From the basic invariance relation

$$[t^a, Y^a] = Y^b \theta^A_{ba} \tag{A.1}$$

which is required by invariance of the Yukawa couplings under the group G, it is possible to derive a number of relations between the group theoretic factors which appear in the various diagrams.

Some relations relevant to the g^4Y terms are

$$t^A t^B Y^a t^B t^A = \tfrac{1}{4}\{C_2(F),\{C_2(F),Y^a\}\} - \tfrac{1}{2}C_2(S)\{C_2(F),Y^a\} + \tfrac{1}{4}[C_2(S)]^2 Y^a \tag{A.2}$$

$$t^A t^B Y^a t^A t^B = \tfrac{1}{4}\{C_2(F),\{C_2(F),Y^a\}\} - \tfrac{1}{4}[2C_2(S) + C_2(G)]\{C_2(F),Y^a\} + \tfrac{1}{4}C_2(S)[C_2(S) + C_2(G)]Y^a \tag{A.3}$$

$$\theta^A_{ab}\theta^B_{bc} Y^c t^B t^A + t^A t^B Y^c \theta^A_{cb}\theta^B_{ba} = \tfrac{1}{2}[C_2(F),[C_2(F),Y^a]] + \tfrac{1}{2}[C_2(S)]^2 Y^a \tag{A.4}$$

$$\theta^A_{ab}\theta^B_{bc} Y^c t^A t^B + t^A t^B Y^c \theta^A_{cb}\theta^B_{ba} = \tfrac{1}{2}[C_2(F),[C_2(F),Y^a]] + \tfrac{1}{2}C_2(S)[C_2(S) - C_2(G)]Y^a \tag{A.5}$$

$$\theta^A_{ab}\theta^B_{bc} t^A Y^c t^B = \theta^A_{ab}\theta^B_{bc} t^B Y^c t^A = \tfrac{1}{4}[C_2(F),[C_2(F),Y^a]] - \tfrac{1}{4}[C_2(S)]^2 Y^a \tag{A.6}$$

Relevant to the g^2Y^3 terms are the relations

$$t^A Y^b Y^{\dagger a} Y^b t^A = \tfrac{1}{2}\{C_2(F), Y^b Y^{\dagger a} Y^b\} - \tfrac{1}{2}C_2^{ac}(S) Y^b Y^{\dagger c} Y^b \tag{A.7}$$

$$t^A Y^b Y^{\dagger a} t^A Y^b + Y^b t^A Y^{\dagger a} Y^b t^A = \tfrac{1}{2}\{C_2(F), Y^b Y^{\dagger a} Y^b\} + \tfrac{1}{2}Y^b\{C_2(F), Y^{\dagger a}\}Y^b - C_2^{ac}(S) Y^b Y^{\dagger c} Y^b - C_2^{bc}(S) Y^b Y^{\dagger a} Y^c \tag{A.8}$$

$$\theta^A_{ac}[t^A, Y^b Y^{\dagger c} Y^b] = -\, C_2^{ac}(S) Y^b Y^{\dagger c} Y^b \tag{A.9}$$

One invariant which appears in the β-function but which does not seem to be reducible to terms involving only Casimir operators is

$$t^A Y^a Y^{\dagger b} t^A Y^b + Y^b t^A Y^{\dagger b} Y^a t^A = \tfrac{1}{2}\{C_2(F), Y^a\} Y_2(F) + \tfrac{1}{2}Y_2^{\dagger}(F)\{C_2(F), Y^a\} - \tfrac{1}{2}C_2^{ac}(S)[Y^c Y_2(F) + Y_2^{\dagger}(F) Y^c] - \theta^A_{bc} t^A Y^a Y^{\dagger b} Y^c - \theta^A_{cb} Y^c Y^{\dagger b} Y^a t^A \tag{A.10}$$

Further relations are given in Appendix A of I.

APPENDIX B: Minimal SU(3) x SU(2) x U(1) Model

In the minimal version of the standard SU(3) x SU(2) x U(1) QCD-electroweak theory, the left-handed fermions consist of quark and lepton doublets denoted by q and ℓ, respectively and antiquark and antilepton singlets denoted by $\bar{u}$, $\bar{d}$, $\bar{e}$ in some number $n_g \geq 3$ of generations. The Yukawa and scalar quartic couplings of the single scalar SU(2) doublet ϕ can be written as

$$\mathcal{L}_{int} = -\bar{e}F_L\phi^\dagger\ell - \bar{d}F_D\phi^\dagger q - \bar{u}H\phi^{\dagger c}q + \text{H.C.} - \frac{1}{2}\lambda(\phi^\dagger\phi)^2 \tag{B.1}$$

where $\phi^c = i\tau_2\phi^*$ is the conjugate scalar doublet, and F_L, F_D, H are matrices in generation space. Here, as in I, we follow the notation of [10] except for the interchange $H \leftrightarrow H^\dagger$.

To present our results in a reasonably concise form, we first write the two-loop evolution equation for the Yukawa coupling matrices in the form

$$H^{-1}\frac{dH}{dt} = \frac{1}{16\pi^2}\beta_U^{(1)} + \frac{1}{(16\pi^2)^2}\beta_U^{(2)} \tag{B.2}$$

$$F_{D,L}^{-1}\frac{dF_{D,L}}{dt} = \frac{1}{16\pi^2}\beta_{D,L}^{(1)} + \frac{1}{(16\pi^2)^2}\beta_{D,L}^{(2)} \tag{B.3}$$

where $\beta^{(1)}$, $\beta^{(2)}$ denote the one-loop and two-loop contributions, respectively.

We recall from [10] that

$$\beta_U^{(1)} = \frac{3}{2}(H^\dagger H - F_D^\dagger F_D) + Y_2(S) - \left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right) \tag{B.4}$$

$$\beta_D^{(1)} = \frac{3}{2}(F_D^\dagger F_D - H^\dagger H) + Y_2(S) - \left(\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right) \tag{B.5}$$

$$\beta_L^{(1)} = \frac{3}{2}F_L^\dagger F_L + Y_2(S) - \frac{9}{4}(g_1^2 + g_2^2) \tag{B.6}$$

where

$$Y_2(S) = \text{Tr}(3H^\dagger H + 3F_D^\dagger F_D + F_L^\dagger F_L) \tag{B.7}$$

and g_1, g_2, g_3 are the U(1), SU(2) and SU(3) gauge couplings, respectively, with a standard normalization.

The two-loop results can be written as

$$\begin{aligned}\beta_U^{(2)} = {} & \frac{3}{2}(H^\dagger H)^2 - H^\dagger H F_D^\dagger F_D - \frac{1}{4}F_D^\dagger F_D H^\dagger H + \frac{11}{4}(F_D^\dagger F_D)^2 + Y_2(S)\left(\frac{5}{4}F_D^\dagger F_D - \frac{9}{4}H^\dagger H\right) \\ & - \chi_4(S) + \frac{3}{2}\lambda^2 - 2\lambda(3H^\dagger H + F_D^\dagger F_D) + \left(\frac{223}{80}g_1^2 + \frac{135}{16}g_2^2 + 16g_3^2\right)H^\dagger H \\ & - \left(\frac{43}{80}g_1^2 - \frac{9}{16}g_2^2 + 16g_3^2\right)F_D^\dagger F_D + \frac{5}{2}Y_4(S) + \left(\frac{9}{200} + \frac{29}{45}n_g\right)g_1^4 - \frac{9}{20}g_1^2g_2^2 \\ & + \frac{19}{15}g_1^2g_3^2 - \left(\frac{35}{4} - n_g\right)g_2^4 + 9g_2^2g_3^2 - \left(\frac{404}{3} - \frac{80}{9}n_g\right)g_3^4\end{aligned} \tag{B.8}$$

and

$$\bar{\beta}^{(2)}_{D} = \frac{3}{2}(F_D^\dagger F_D)^2 - F_D^\dagger F_D H^\dagger H - \frac{1}{4}H^\dagger H F_D^\dagger F_D + \frac{11}{4}(H^\dagger H)^2$$

$$+ Y_2(S)(\frac{5}{4}H^\dagger H - \frac{9}{4}F_D^\dagger F_D) - X_4(S) + \frac{3}{2}\lambda^2 - 2\lambda(3F_D^\dagger F_D + H^\dagger H)$$

$$+(\frac{187}{80}g_1^2 + \frac{135}{16}g_2^2 + 16g_3^2)F_D^\dagger F_D - (\frac{79}{80}g_1^2 - \frac{9}{16}g_2^2 + 16g_3^2)H^\dagger H \quad \text{(B.9)}$$

$$+ \frac{5}{2}Y_4(S) - (\frac{29}{200} + \frac{1}{45}n_g)g_1^4 - \frac{27}{20}g_1^2 g_2^2 + \frac{31}{15}g_1^2 g_3^2$$

$$-(\frac{35}{4} - n_g)g_2^4 + 9g_2^2 g_3^2 - (\frac{404}{3} - \frac{80}{9}n_g)g_3^4$$

and, finally,

$$\bar{\beta}^{(2)}_{L} = \frac{3}{2}(F_L^\dagger F_L)^2 - \frac{9}{4}Y_2(S)F_L^\dagger F_L - X_4(S) + \frac{3}{2}\lambda^2 - 6\lambda F_L^\dagger F_L$$

$$+ (\frac{387}{80}g_1^2 + \frac{135}{16}g_2^2)F_L^\dagger F_L + \frac{5}{2}Y_4(S) + (\frac{51}{200} + \frac{11}{5}n_g)g_1^4 \quad \text{(B.10)}$$

$$+ \frac{27}{20}g_1^2 g_2^2 - (\frac{35}{4} - n_g)g_2^4$$

Here $Y_2(S)$ has been defined by Eq. (B.7). The invariant $Y_4(S)$ is defined by

$$\mathrm{Tr}\{C_2(F), Y^c\}Y^b = Y_4(S)\delta^{ab} \quad \text{(B.11)}$$

and

$$Y_4(S) = (\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2)\mathrm{Tr}H^\dagger H + (\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2)\mathrm{Tr}F_D^\dagger F_D$$

$$+ \frac{3}{4}(g_1^2 + g_2^2)\mathrm{Tr}F_L^\dagger F_L \quad \text{(B.12)}$$

The invariant $X_4(S)$ is defined by

$$\frac{1}{2}\mathrm{Tr}\{\frac{3}{2}[Y_2(F)Y^{\dagger b} + Y^{\dagger b}Y_2(F)]Y^a + 2Y^{\dagger b}Y^c Y^{\dagger a}Y^c\} \equiv X_4(S)\delta^{ab} \quad \text{(B.13)}$$

and

$$X_4(S) = \frac{9}{4}\mathrm{Tr}\{3(H^\dagger H)^2 + 3(F_D^\dagger F_D)^2 + (F_L^\dagger F_L)^2 - \frac{2}{3}H^\dagger H F_D^\dagger F_D\} \quad \text{(B.14)}$$

If mixing between generations is neglected and the scalar quartic coupling is dropped, these results reduce to those of Fischler and Oliensis [11], apart from an obvious misprint, and our results also agree diagram by diagram with calculations recently given by the same authors [9].

References

[1] H. E. Machacek and M. T. Vaughn, Nucl. Phys. B222 (1983) 83.

[2] G. 't Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189.

[3] W. A. Bardeen, A. J. Buras, D. W. Duke, and T. W. Muta, Phys. Rev. D18 (1978) 3998.

[4] K. Fujikawa, B. W. Lee and A. I. Sanda, Phys. Rev. D6 (1972) 2973; E. S. Abers and B. W. Lee, Phys. Reports 9C (1973) 1.

[5] G. 't Hooft, Nucl. Phys. B61 (1963) 455; J. C. Collins and A. J. MacFarlane, Phys. Rev. D10 (1974) 1201.

[6] T. P. Cheng, E. Eichten, and L. -F. Li, Phys. Rev. D9 (1974) 2259.

[7] M. T. Vaughn, Z. Phys. C13 (1982) 149.

[8] A. A. Vladimirov, Teor. Mat. Fiz. 25 (1975) 335.

[9] M. Fischler and J. Oliensis, Fermilab preprint 83/84-THY (1983).

[10] H. E. Machacek and M. T. Vaughn, Phys. Lett 103B [illegible] 427.

[11] M. Fischler and J. Oliensis, Phys. Lett. 119B (1982) 385.

TABLE I

Singular parts of the vertex renormalization Z^a, as defined by Eq. (2.8), for the one-loop propagator insertions in the one-loop vertex diagram of Fig. 1, with insertions as designated in Fig. 3(a).

Diagram	S^a	A	B
(1/P1)	$\kappa Y_2^{bc}(S) Y^b Y^{\dagger a} Y^c$	-1	1
	$C_2^{bc}(S) Y^b Y^{\dagger a} Y^c$	$1 + \frac{1}{2}\alpha$	$-1 + \frac{1}{2}\alpha$
(1/P2) + (1/P3)	$Y^b Y^{\dagger a} Y_2^{\dagger}(F) Y^b + Y^b Y_2(F) Y^{\dagger a} Y^b$	$-\frac{1}{4}$	$\frac{1}{4}$
	$Y^b \{C_2(F), Y^{\dagger a}\} Y^b$	$-\frac{1}{2}(1 - \alpha)$	0
(2/P1)	$t^A Y^a t^A$	Eq. (2.10)	Eq. (2.11)
(2/P2) + (2/P3)	$t^A Y^a t^A Y_2(F) + Y_2^{\dagger}(F) t^A Y^a t^A$	$1 - \frac{1}{4}\alpha$	$-\frac{3}{2} + \frac{1}{4}\alpha$
	$\{C_2(F), t^A Y^a t^A\}$	$\frac{1}{2}(1 - \alpha)(4 - \alpha)$	$-(1 - \alpha)$
(3a/P1) + (3b/P1)	$\theta^A_{ab} t^A Y_2^{\dagger}(F) Y^b - \theta^A_{ab} Y^b Y_2(F) t^A$	$-\frac{1}{4}(1 - \alpha)$	$\frac{1}{4}(1 - \alpha)$
	$\theta^A_{ab}[C_2(F) t^A, Y^b]$	$-\frac{1}{2}(1 - \alpha)^2$	0

TABLE I (continued)

Diagram	$\underset{\sim}{S}^a$	A	B
(3a/P2) + (3b/P3)	$\kappa\theta^A_{ab}Y^{bc}_2(S)[\underset{\sim}{t}^A,\underset{\sim}{Y}^c]$	$-(1-\alpha)$	$1-\alpha$
	$\theta^A_{ab}C^{bc}_2(S)[\underset{\sim}{t}^A,\underset{\sim}{Y}^c]$	$\frac{1}{2}(1-\alpha)(2+\alpha)$	$-\frac{1}{2}(1-\alpha)(2-\alpha)$
(3a/P3) + (3b/P2)	---	0	0

TABLE II

Singular parts of the vertex renormaliztion $\underset{\sim}{Z}^a$, as defined by Eq. (2.8), for the one-loop vertex insertions in the one-loop vertex diagrams of Fig. 1, with insertions as designated in Fig. 3(b).

Diagram	$\underset{\sim}{S}^a$	A	B
(1/V1)	$\underset{\sim}{Y}^c\underset{\sim}{Y}^{\dagger b}\underset{\sim}{Y}^a\underset{\sim}{Y}^{\dagger b}\underset{\sim}{Y}^c$	$-\frac{1}{2}$	$\frac{1}{2}$
	$\underset{\sim}{Y}^b\{\underset{\sim}{C}_2(F),\underset{\sim}{Y}^{\dagger a}\}\underset{\sim}{Y}^b$	$1-\frac{1}{4}\alpha$	$-\frac{1}{4}(2-\alpha)$
	$C^{ac}_2(S)\underset{\sim}{Y}^b\underset{\sim}{Y}^{\dagger c}\underset{\sim}{Y}^b$	$-\frac{1}{4}(2+\alpha)$	$\frac{1}{4}(2-\alpha)$
(1/V2) + (1/V3)	$\underset{\sim}{Y}^b\underset{\sim}{Y}^{\dagger c}\underset{\sim}{Y}^b\underset{\sim}{Y}^{\dagger a}\underset{\sim}{Y}^c + \underset{\sim}{Y}^c\underset{\sim}{Y}^{\dagger a}\underset{\sim}{Y}^b\underset{\sim}{Y}^{\dagger c}\underset{\sim}{Y}^b$	$-\frac{1}{2}$	0
	$\{\underset{\sim}{C}_2(F),\underset{\sim}{Y}^b\underset{\sim}{Y}^{\dagger a}\underset{\sim}{Y}^b\}$	$1-\frac{1}{4}\alpha$	$\frac{5}{4}$
	$\underset{\sim}{Y}^b\{\underset{\sim}{C}_2(F),\underset{\sim}{Y}^{\dagger a}\}\underset{\sim}{Y}^b$	$1-\frac{1}{4}\alpha$	$-\frac{1}{4}$
	$C^{bc}_2(S)\underset{\sim}{Y}^b\underset{\sim}{Y}^{\dagger a}\underset{\sim}{Y}^c$	$-(1+\frac{1}{2}\alpha)$	$-(\frac{3}{2}+\alpha)$
(2/V1)	$\underset{\sim}{t}^A\underset{\sim}{Y}^b\underset{\sim}{Y}^{\dagger a}\underset{\sim}{Y}^b\underset{\sim}{t}^A$	$1-\frac{1}{4}\alpha$	$-3+\frac{1}{2}\alpha$
	$\underset{\sim}{t}^A\{\underset{\sim}{C}_2(F),\underset{\sim}{Y}^a\}\underset{\sim}{t}^A$	$-\frac{1}{4}(4-\alpha)^2$	$\frac{1}{4}(4-\alpha)^2$
	$C^{ab}_2(S)\underset{\sim}{t}^A\underset{\sim}{Y}^b\underset{\sim}{t}^A$	$\frac{1}{4}(4-\alpha)(2+\alpha)$	$-\frac{1}{4}(12-4\alpha+\alpha^2)$

TABLE II (continued)

Diagram	S^a	A	B
(2/V2)	$Y_2^\dagger(F)\,t^A Y^b t^A$	$-(1 - \frac{1}{4}\alpha)$	$\frac{1}{2}(3 - \frac{1}{2}\alpha)$
+ (2/V3)	$+\; t^A Y^a t^A Y_2(F)$		
	$t^A Y^a Y^{\dagger b} t^A Y^b$	0	$-\frac{3}{2}$
	$+\; Y^b t^A Y^{\dagger b} Y^a t^A$		
	$\{C_2(F), t^A Y^a t^A\}$	$-\frac{1}{2}(1 - \alpha)(4 - \alpha)$	$-(2 + \alpha)$
	$C_2(G)\,t^A Y^a t^A$	$-\frac{1}{4}(4 - \alpha)^2$	$\frac{1}{2}(2 + \alpha)(7 - \alpha)$
(3a/V1)	$\kappa\theta^A_{ab} Y_2^{bc}(S)\,[t^A, Y^c]$	$1 - \alpha$	$-(1 - \alpha)$
+ (3b/V1)	$\theta^A_{ab} C_2^{bc}(S)\,[t^A, Y^c]$	$-\frac{1}{2}(1 - \alpha)(2 + \alpha)$	$\frac{1}{2}(1 - \alpha)(2 - \alpha)$
	$C_2(G)\,\theta^A_{ab}[t^A, Y^b]$	$\frac{1}{8}(1 - \alpha)(4 - \alpha)$	$-\frac{1}{8}(1 - \alpha)(2 - \alpha)$
(3a/V2)	$\theta^A_{ab} Y_2^\dagger(F)\,t^A Y^b$	$\frac{1}{4}(1 - \alpha)$	$-\frac{1}{4}(1 - \alpha)$
+ (3b/V3)	$-\theta^A_{ab} Y^b t^A Y_2(F)$		
	$\theta^A_{ab} C_2(F)\,t^A Y^b$	$\frac{1}{2}(1 - \alpha)^2$	0
	$-\theta^A_{ab} Y^b t^A C_2(F)$		
	$C_2(G)\,\theta^A_{ab}[t^A, Y^b]$	$\frac{1}{8}(1 - \alpha)(4 - \alpha)$	$-\frac{1}{8}(1 - \alpha)(2 - \alpha)$

TABLE II (continued)

Diagram	S^a	A	B
(3a/V3)	$\theta^A_{ab}[t^A, Y^c Y^{\dagger b} Y^c]$	$-\frac{1}{2}(1 - \alpha)$	0
+ (3b/V2)	$\theta^A_{ab} C_2(F)\,t^A Y^b$	$\frac{1}{4}(1 - \alpha)(4 - \alpha)$	$-\frac{1}{4}(1 - \alpha)$
	$-\theta^A_{ab} Y^b t^A C_2(F)$		
	$\theta^A_{ab} t^A Y^b C_2(F)$	$\frac{1}{4}(1 - \alpha)(4 - \alpha)$	$\frac{5}{4}(1 - \alpha)$
	$-\theta^A_{ab} C_2(F)\,Y^b t^A$		
	$\theta^A_{ab} C_2^{bc}(S)\,[t^A, Y^c]$	$-\frac{1}{4}(1 - \alpha)(2 + \alpha)$	$-\frac{1}{4}(1 - \alpha)(3 + 2\alpha)$

TABLE III

Singular parts of the vertex renormalization $\underset{\sim}{Z}^a$, as defined by Eq. (2.8), for the crossed ladder topology diagrams of Fig. 2. These diagrams have at most a single pole in ε, so the coefficient A = 0 in each case.

Diagram	β^a	B
(1)	$\lambda_{abcd}\underset{\sim}{Y}^b\underset{\sim}{Y}^{\dagger c}\underset{\sim}{Y}^d$	$\frac{1}{2}$
(2)	$\underset{\sim}{Y}^c\underset{\sim}{Y}^{\dagger b}\underset{\sim}{Y}^a\underset{\sim}{Y}^{\dagger c}\underset{\sim}{Y}^b$	$-\frac{1}{2}$
(3a) + (3b)	$\underset{\sim}{Y}^b\underset{\sim}{t}^A\underset{\sim}{Y}^{\dagger a}\underset{\sim}{Y}^b\underset{\sim}{t}^A + \underset{\sim}{t}^A\underset{\sim}{Y}^b\underset{\sim}{Y}^{\dagger a}\underset{\sim}{t}^A\underset{\sim}{Y}^b$	$-(1 + \frac{1}{2}\alpha)$
(4)	$\underset{\sim}{t}^B\underset{\sim}{t}^A\underset{\sim}{Y}^a\underset{\sim}{t}^B\underset{\sim}{t}^A$	$-(2 - 4\alpha + \frac{1}{2}\alpha^2)$
(5)	$\theta^A_{ad}\theta^A_{bc}\underset{\sim}{Y}^b\underset{\sim}{Y}^{\dagger d}\underset{\sim}{Y}^c$	0
(6)	$-if^{ABC}\theta^C_{ab}\underset{\sim}{t}^B\underset{\sim}{Y}^b\underset{\sim}{t}^A$	0
(7a) + (7b)	$(\underset{\sim}{\theta}^A\underset{\sim}{\theta}^B)_{ab}\underset{\sim}{Y}^b\underset{\sim}{t}^A\underset{\sim}{t}^B + \underset{\sim}{t}^A\underset{\sim}{t}^B\underset{\sim}{Y}^b(\underset{\sim}{\theta}^A\underset{\sim}{\theta}^B)_{ba}$	$-\frac{3}{2}(1 - \alpha)$
(8a) + (8b)	$\{\underset{\sim}{\theta}^A,\underset{\sim}{\theta}^B\}_{ab}\underset{\sim}{Y}^b\underset{\sim}{t}^A\underset{\sim}{t}^B + \underset{\sim}{t}^A\underset{\sim}{t}^B\underset{\sim}{Y}^b\{\underset{\sim}{\theta}^A,\underset{\sim}{\theta}^B\}_{ba}$	$1 - \frac{1}{2}\alpha + \frac{1}{4}\alpha^2$
(9)	$\{\underset{\sim}{\theta}^A,\underset{\sim}{\theta}^B\}_{ab}\underset{\sim}{t}^A\underset{\sim}{Y}^b\underset{\sim}{t}^B$	$-(2 - \alpha + \frac{1}{2}\alpha^2)$

Figure Captions

Fig. 1 One-loop corrections to the proper Yukawa vertex.

Fig. 2 Two-loop corrections to the proper Yukawa vertex with crossed ladder topology.

Fig. 3 Notation for (a) propagator insertions, and (b) vertex insertions in the one-loop corrections to the proper Yukawa vertex.

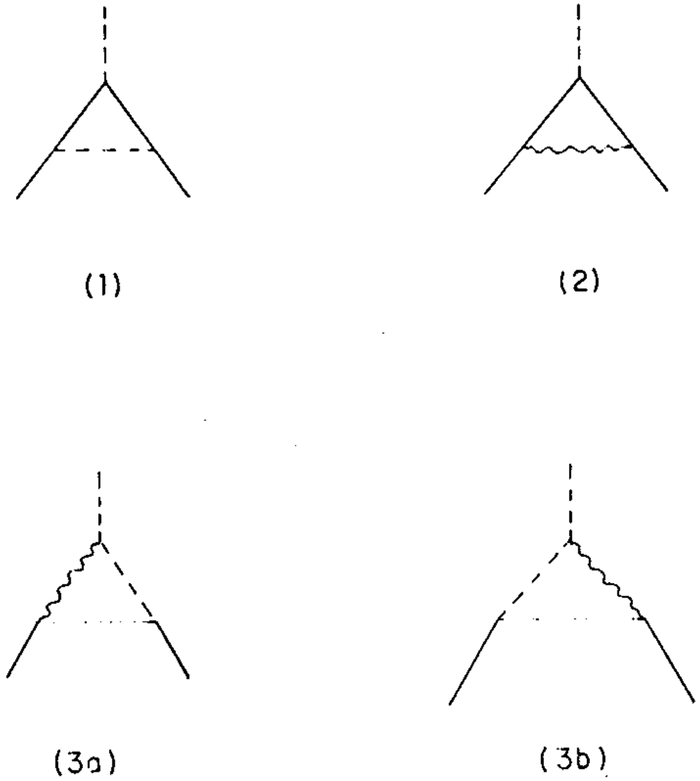

Figure 1

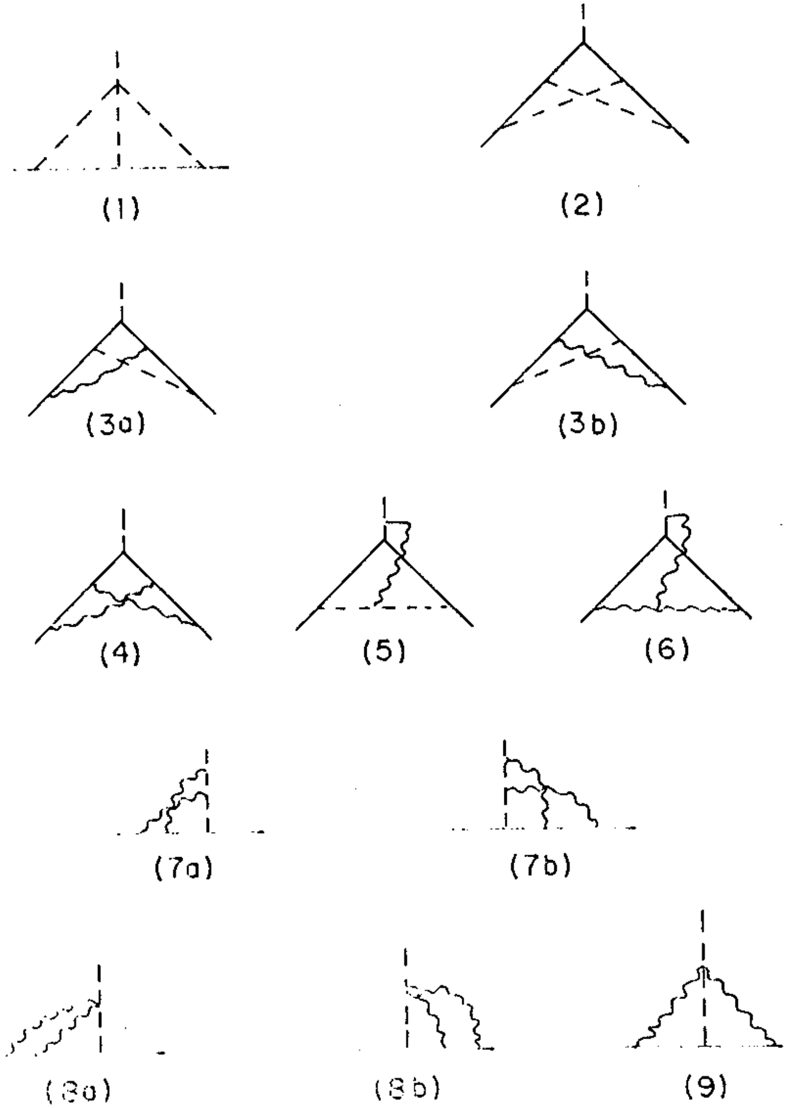

Figure 2

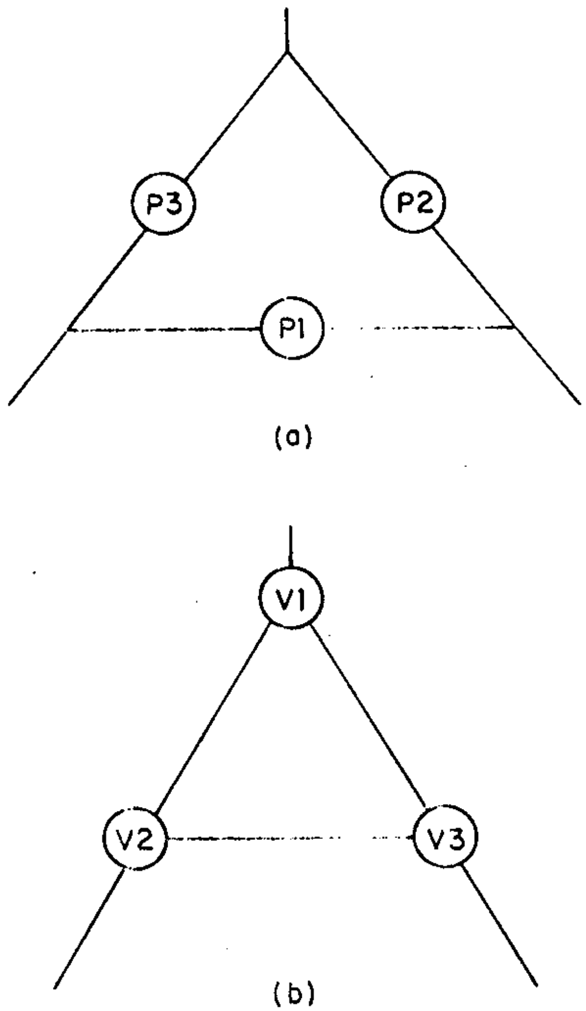

Figure 3