

PHY2049 Fall 2000 EXAM 4

1. A transformer with 1000 primary turns and 250 secondary turns is connected to a $100\ \Omega$ resistor. If a 220 V ac power supply is connected to the primary set of turns, what is the current in the secondary circuit? **Answer: 0.55 A**

The ratio of the voltages is equal to the ratio of the turns. The voltage in the secondary is

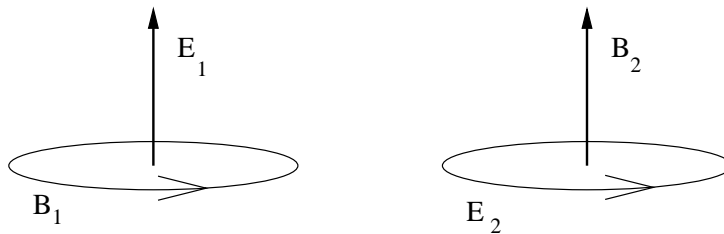
$$\begin{aligned}\frac{V_s}{V_p} &= \frac{N_s}{N_p} \\ V_s &= V_p \frac{N_s}{N_p} \\ &= 220\ \text{V} \left(\frac{250}{1000} \right) \\ &= 55\ \text{V}.\end{aligned}$$

The current is $I_s = V_s/R_s = (55\ \text{V})/(100\ \Omega) = 0.55\ \text{A}$.

2. The electric field of an electromagnetic wave has the magnitude of 500 V/m. What is the magnitude of the corresponding magnetic field in the electromagnetic wave? **Answer: $1.7 \times 10^{-7}\ \text{T}$**

The amplitude of the magnetic field is related to the electric field amplitude $B = E/c = (500\ \text{V/m})/(3.00 \times 10^8\ \text{m/s}) = 1.7 \times 10^{-6}\ \text{T}$.

3. Electric field E_1 changes and, thus, induces magnetic field B_1 . The changing magnetic field B_2 induces electric field E_2 . Given the figure shown below, can you judge how magnitudes of electric field E_1 and magnetic field B_2 change? **Answer: Electric field E_1 increases and magnetic field B_2 decreases**



Maxwell's law is needed to interpret the first diagram and Faraday's law is needed for the second. Maxwell's law states that a changing electric flux induces a magnetic field and that the induced magnetic field supports the change that created it. The first diagram shows that when the fingers of the right hand curl in the direction of B_1 , the thumb points in the direction of E_1 . Since this is consistent with Maxwell's law, the value of E_1 has increased. The second diagram is also consistent with the right-hand rule. However, the Lenz's law portion of Faraday's law says that the induced electric field opposes the changing

magnetic flux that created it. Lenz's law states that the right-hand rule is opposed and B_2 has decreased in the second diagram.

4. A linearly polarized wave of wavelength 300 m propagates in z-direction. At $z=0$ and $t=0$, the electric field assumes its maximum value E_0 along the positive y-direction. At $t=0.5 \mu\text{s}$, amplitude of the electric field is: **Answer: $-E_0$**

The most general expression for the electric field component of a traveling electromagnetic wave propagating in the $+z$ direction is

$$E(z, t) = E_0 \sin(kz - \omega t + \phi)$$

It is given that $E(0, 0) = E_0$. Substituting

$$E_0 = E_0 \sin(0 - 0 + \phi)$$

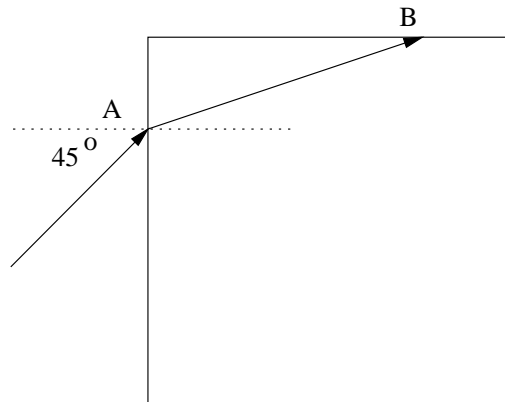
$$\sin \phi = 1$$

$$\phi = \pi/2.$$

The wavenumber is $k = 2\pi/\lambda = 2\pi/(300 \text{ m}) = 0.0209 \text{ 1/m}$. The frequency is $\omega = ck = (3.00 \times 10^8)(0.0209 \text{ 1/m}) = 6.28 \times 10^6 \text{ rad/s}$. At $z = 0$ and $t = 0.5 \mu\text{s}$, the electric field is

$$E = E_0 \sin[k(0) - (6.28 \times 10^6 \text{ rad/s})(0.5 \times 10^{-6} \text{ s}) + \pi/2] = -E_0.$$

5. A ray of light enters a glass cube at point A at an incident angle of 45° and then undergoes total internal reflection at point B. What minimum value of index of reflection n of glass can be inferred from this information? **Answer: 1.22**



Applying Snell's law at the first interface

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1 \sin 45^\circ = n \sin \theta_2$$

$$\sin \theta_2 = \frac{1}{\sqrt{2} n}.$$

For total internal reflection to occur at the other surface, $90^\circ - \theta_2 \geq \theta_c$. Taking the sine of both sides, $\sin(90^\circ - \theta_2) \geq \sin \theta_c$. The critical angle is given by $\sin \theta_c = 1/n$. This leads to

$$\begin{aligned} \sin(90^\circ - \theta_2) &\geq \sin \theta_c \\ \cos \theta_2 &\geq \sin \theta_c \\ \cos^2 \theta_2 &\geq \sin^2 \theta_c \\ 1 - \sin^2 \theta_2 &\geq \frac{1}{n^2} \\ 1 - \frac{1}{(\sqrt{2} n)^2} &\geq \frac{1}{n^2} \\ 2n^2 - 1 &\geq 2 \\ n^2 &\geq 3/2 \\ n &\geq \sqrt{3/2} = 1.22. \end{aligned}$$

6. An LCR circuit has the following elements: $L=1$ mH, $C=2$ μ F, $R=2$ Ω . What is the resonance frequency f_0 of this circuit? (Note that $2\pi f_0 = \omega_0$.) **Answer: 3.6 kHz**

The resonant frequency of an *RLC* circuit is

$$\begin{aligned} \omega' &= \sqrt{\omega^2 - (R/2L)^2} \\ &= \sqrt{1/LC - (R/2L)^2} \\ &= \sqrt{1/((1.0 \times 10^{-3} \text{ H})(2.0 \times 10^{-6} \text{ F})) - ((2.0 \Omega)/(2(1.0 \times 10^{-3} \text{ H})))} \\ &= 2.23 \times 10^4 \text{ rad/s.} \end{aligned}$$

The corresponding linear frequency is $f = \omega/2\pi = 3600$ Hz.

7. Three people D, F, and P have identical solenoids (of equal inductance). They also have identical in size rods of different materials: the person D has diamagnetic rod, the F – ferromagnetic, the person P – paramagnetic. All three of them insert their rods inside a solenoid. How would the inductances of these solenoids change? Order from smallest to the largest. **Answer: DPF**

Before the material is inserted into the solenoid, there is a certain magnetic field inside the solenoid, B_0 , and a certain magnetic flux, Φ_0 . When the diamagnetic material is inserted, the magnetic field in the solenoid is reduced to B_D and the magnetic flux is reduced to Φ_D . However, when the paramagnetic material is inserted, the magnetic field is increased to B_P and the magnetic flux is increased to Φ_P . And when the ferromagnetic material is inserted, the magnetic field will be greatly increased to B_F and the magnetic flux also greatly increased to Φ_F . The magnetic fluxes in the solenoids are ordered as $\Phi_D < \Phi_P < \Phi_F$. Inductance is defined as $L = N\Phi/I$, giving $\Phi = LI/N$. This implies, $L_D < L_P < L_F$.

8. There have been serious proposals to propel a spacecraft across a solar system by using radiation pressure on a huge sail made of some very thin metallized material (perfect

reflector). Suppose that the mass of the spacecraft is $m=1500$ kg. The only meaningful source of radiation is the sun; for simplicity assume that the sail is perpendicular to the sun's radiation. Calculate the area of the sail needed in order that, at any distance from the sun, the radiation force of the sail is equal in magnitude to the gravitational attraction of the sun on the spacecraft. (Needed data: Sun's mass is $M=2 \times 10^{30}$ kg, Power emitted by the sun is $P=4 \times 10^{26}$ W; Universal gravitational constant is $G=6.7 \times 10^{-11}$ N·m²/kg²; speed of light in vacuum is $c=3 \times 10^8$ m/s). **Answer: 0.95 km²**

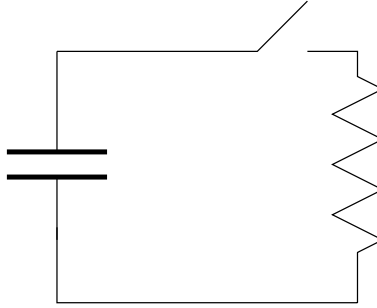
The radiation pressure on a perfect reflector is $p_r = 2I/c$ and the intensity is the power per unit area, $I=\text{Power}/\text{Area}=P/4\pi r^2$. This pressure balances out the gravitation attraction of the sun.

$$\begin{aligned}
 p_r A &= F_G \\
 \frac{2I}{c} A &= F_G \\
 \frac{2P}{4\pi r^2 c} A &= \frac{GmM}{r^2} \\
 A &= \frac{2\pi GmMc}{P} \\
 &= \frac{2\pi(6.7 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(1500 \text{ kg})(2 \times 10^{30} \text{ kg})(3 \times 10^8 \text{ m/s})}{4 \times 10^{26} \text{ W}} \\
 &= 9.5 \times 10^5 \text{ m}^2 \times \left(\frac{1 \text{ km}}{1000 \text{ m}} \right)^2 \\
 &= 0.95 \text{ km}.
 \end{aligned}$$

9. You are asked to design a good tuner that would provide as clear as possible reception of a particular AM station. The antenna of the tuner is to be connected to a resonant LCR circuit. How would you try to optimize this circuit? **Answer: maximize L and minimize R**

To receive the largest signal, the circuit should give a large current at resonance. At resonance the current is $i = \mathcal{E}/R$. To maximize the current, the resistance should be minimized. Also, to select only the desired station, the resonance peak should be very narrow. Since the peak width is $\Delta\omega = R/2L$ and R was minimized to increase the signal, the inductance must be maximized.

10. A parallel plate capacitor made of two disks of radius $r=1$ cm separated by a distance $d=1$ mm is charged to $Q_0=1 \mu\text{C}$. Then, the switch is closed and the capacitor discharges through a resistor $R=1 \Omega$. What is the maximal magnetic field between the plates of the capacitor? *Hints: Calculate the change of the electric field between plates and use one of the Maxwell's equations that relates an induced magnetic field and a changing electric field flux. Alternatively, you can make use of the displacement current notion. Also, recall that a parallel plate capacitor has capacitance $C = A\epsilon_0/d$, and that a charge on a capacitor in an RC-circuit decays exponentially as $Q(t) = Q_0 e^{-t/\tau}$, where $\tau = RC$.* **Answer: 7.2 T**



The Ampere-Maxwell law states

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}.$$

In the gap between the plates of the capacitor, $I_{enc} = 0$. The electric field in the capacitor is related to the surface charge density σ , $E = \sigma/\epsilon_0$. The surface charge density is given by $\sigma = Q/A$. The electric flux is $\Phi_E = EA = (\sigma/\epsilon_0)(A) = (Q/\epsilon_0 A)(A) = Q/\epsilon_0$. The Ampere-Maxwell law becomes

$$\oint \vec{B} \cdot d\vec{s} = 0 + \mu_0 \epsilon_0 \frac{1}{\epsilon_0} \frac{dQ}{dt} = \mu_0 \frac{dQ}{dt}.$$

The largest magnetic field will be at the edge of the capacitor where the entire electric flux is enclosed by the integration contour. Integrating on this contour

$$\oint \vec{B} \cdot d\vec{s} = B2\pi r.$$

The right side of the Ampere-Maxwell law is evaluated

$$\begin{aligned} \mu_0 \frac{dQ}{dt} &= \mu_0 \frac{d}{dt} (Q_0 e^{-t/\tau}) \\ &= -\frac{\mu_0 Q_0}{\tau} e^{-t/\tau}. \end{aligned}$$

Substituting into the Ampere-Maxwell law (and dropping the minus sign since only the magnitude of B is sought)

$$\begin{aligned} B2\pi r &= \frac{\mu_0 Q_0}{\tau} e^{-t/\tau} \\ &= \frac{\mu_0 Q_0}{RC} e^{-t/\tau} \\ B &= \frac{\mu_0 Q_0}{2\pi r RC} e^{-t/\tau} \\ B_{max} &= \frac{\mu_0 Q_0}{2\pi r RC} \end{aligned}$$

and the largest magnetic field will occur when $t = 0$. The only remaining unknown is the value of the capacitance.

$$C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 \pi r^2}{d} = \frac{(8.85 \times 10^{-12} \text{ F/m}) \pi (0.01 \text{ m})^2}{1.0 \times 10^{-3} \text{ m}} = 2.78 \times 10^{-12} \text{ F}$$

Finally,

$$B_{max} = \frac{\mu_0 Q_0}{2\pi r RC} = \frac{(4\pi \times 10^{-7} \text{ Tm/A})(1.0 \times 10^{-6} \text{ C})}{2\pi(0.01 \text{ m})(1 \Omega)(2.78 \times 10^{-12} \text{ C})} = 7.20 \text{ T}.$$