

Natural Units

There are 4 primary SI units: three kinematical (meter, second, kilogram) and one electrical (Ampere¹)

It is common in the realm of the elementary particle physics to redefine units so that speed of light and Plank's constant become equal to one: $c=1$ and $\hbar=1$. This imposes two constraints on the three kinematical units and, therefore, leaves us a free choice for one of the three kinematical units. The units of electrical charge, also, can be and are redefined (see below). Such system of units is often referred to as Natural Units (natural for the elementary particle physics, that is).

The kinematical unit of the choice is energy, E, and it is usually measured in eV (keV, MeV, GeV, TeV). Once we fixed $c=1$ and $\hbar=1$, all other kinematical units can now be expressed in terms of units of energy. E.g., one can easily see:

$$E^2 = m^2 c^4 + p^2 c^2: \quad \text{with } c=1, \text{ units of mass and momentum are E}$$

$$\Psi = C e^{-i\left(\frac{Et - px}{\hbar}\right)}: \quad \text{with } \hbar=1, \text{ units of time and length are } 1/E$$

$$L_z = \hbar n: \quad \text{with } \hbar=1, \text{ angular momentum is dimensionless}$$

In SI system, units of current and charge can be *effectively* defined by choosing the value of ϵ_0 in the Coulomb's Law:

$$F = \frac{qq}{r^2} \frac{1}{4\pi\epsilon_0}, \text{ where } \epsilon_0 = 8.8 \times 10^{-12} \text{ C}^2 \text{m}^{-3} \text{kg}^{-1} \text{s}^2.$$

In the Natural Units, the units of charge are defined by choosing $\epsilon_0=1$. This automatically sets $\mu_0=1$, since $c^2=1/(\epsilon_0\mu_0)$. Since the units of force and distance in Natural Units are E^2 and $1/E$, the electric charge turns out to be dimensionless in these units.

The value of the fine structure constant, being dimensionless, is the same in all units. In SI units it has the following form: $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137}$, where e is the elementary charge $e=1.6 \times 10^{-19}$ C. In Natural

Units, the fine structure constant becomes $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$. This clearly shows that an electric charge e has no dimensions in the Natural Units and now equals to 0.303.

Table. Units for major physical quantities in SI and Natural Units with conversion factors.

Quantity	Dimensions		Conversions
	SI Units	Natural Units	
mass	kg	E	1 GeV = 1.8×10^{-27} kg
length	m	1/E	1 GeV ⁻¹ = 0.197×10^{-15} m
time	s	1/E	1 GeV ⁻¹ = 6.58×10^{-25} s
energy	kg·m ² /s ²	E	1 GeV = 1.6×10^{-10} Joules
momentum	kg·m/s	E	1 GeV = 5.39×10^{-19} kg·m/s
velocity	m/s	none	1 = 2.998×10^8 m/s (c)
angular momentum	kg·m ² /s	none	1 = 1.06×10^{-34} J·s (\hbar)
cross-section	m ²	1/E ²	1 GeV ⁻² = 0.389 mb = 0.389×10^{-31} m ²
force	kg·m/s ²	E ²	1 GeV ² = 8.19×10^5 Newton
charge	C=A·s	none	1 = 5.28×10^{-19} Coulomb; $e=0.303=1.6 \times 10^{-19}$ C

¹ Unit of charge, Coulomb, is Ampere · second.

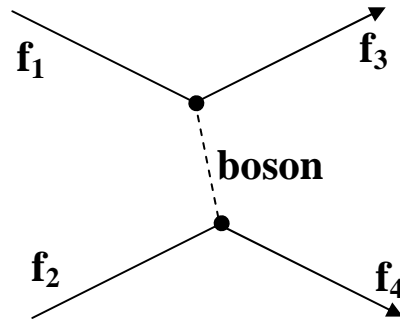
Scales in Natural and SI units to remember

- mass of a proton/neutron $\sim 1 \text{ GeV}$
- mass of an electron $\sim 0.5 \text{ MeV}$

- proton/neutron radius $\sim 1 \text{ fm} = 10^{-15} \text{ m} = 1 \text{ fermi} \sim 1/(200 \text{ MeV})$
- time for light to cross a proton $\sim 10^{-23} \text{ s}$

- highest energy collider (Tevatron) $\sim 2 \text{ TeV}$ center-of-mass energy
- distance scale probed at Tevatron $\sim 1/(2 \text{ TeV}) \sim 10^{-19} \text{ m}$

Periodic Table of Particles/Forces in the Standard Model



Fermions (matter)

ν_e	ν_μ	ν_τ
e^-	μ^-	τ^-
u	c	t
d	s	b

Color charge	Electric charge	Z ⁰ coupling	W [±] coupling	Higgs coupling	Spin	Masses (MeV)		
0	0	yes	yes	yes for m _ν ≠0	1/2	<2 eV	<0.19 MeV	<18 MeV
0	-1	yes	yes	yes	1/2	0.511 MeV	106 MeV	1.777 GeV
r, g, b	+2/3	yes	yes	yes	1/2	3±1.5 MeV	1.2±0.2 GeV	174±5 GeV
r, g, b	-1/3	yes	yes	yes	1/2	7±2 MeV	120±40 MeV	4.3±0.3 GeV

Bosons (forces)

g (8 gluons)	γ (photon)	Z ⁰ boson	W [±] boson	Higgs boson (?)
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Color Charge	8 color-anticolor combinations	0	0	0	0
Electric Charge	0	0	0	±1	0
Spin	1	1	1	1	0
Mass	0	0	91 GeV	80 GeV	>114 GeV
Relative strength*	1	~10 ⁻²	~10 ⁻⁷	~10 ⁻⁷	mass-dependent

Side Note: Boson Self-Coupling Vertices allowed in SM:

- **gluons:** ggg; gggg
- **$\gamma, Z, W:$** $\gamma WW, ZWW;$ $\gamma\gamma WW, \gamma ZWW, ZZWW$
- **H, Z, W:** HZZ, HWW, HHH; HHHH, ZZHH, WWHH

* For quarks at about 1 GeV/c momentum transfers

Fermions: The top set of particles gives $3 \times (2 + 2 \times 3) = 24$ fermions with $\text{spin} = \frac{1}{2}$. They are conventionally called matter. Three columns represent 3 nearly identical generations; the masses being the only difference between them. This three-generation structure allows for not yet understood phenomena of mixing.

Each generation consists of:

- two leptons (charged lepton and an associated neutrino) and
- three pairs of quarks (colored blocks representing quarks with three different color charges)
- all fermions have masses

All particles have their own names:

- electron neutrino and electron, up- and down-quarks;
- muon neutrino and muon, charm- and strange-quarks;
- tau neutrino and tau-lepton, top- and bottom-quarks (*bottom* is also known as *beauty*);

Bosons: There are 5 kinds of bosons. They are mediators of forces:

- 8 gluons ($\text{spin} = 1$) of different color-anticolor charges—responsible for the strong force (coupled to color charge, i.e. only quarks participate in interaction via exchange of gluons)
- 1 photon ($\text{spin} = 1$)—responsible for electromagnetic force (coupled to electric charge, i.e. only charged particles participate in interaction via exchange of photons)
- 1 Z- and 2 W-bosons ($\text{spin} = 1$)—responsible for weak force (coupled to weak hypercharge and weak isospin; all fermions participate in interaction via exchange of Z and W)

NOTE: gluons and photons are massless, Z and W are massive particles

- Higgs ($\text{spin} = 1$)—*the only not yet discovered force/particle*, invented to allow for masses in the weak sector of SM (its coupling to particles is related to their masses: the larger the mass, the larger the force);

Gravity is ignored (it is VERY weak and there is NOT a coherent quantum theory of gravity). Graviton should have zero mass and $\text{spin} = 2$. Its relative strength in the last column should be of the order of 10^{-40} .

In general, fermions—particles with half-integral spin: $\frac{1}{2}, \frac{3}{2}, \dots$. Bosons—particles with integral spin: 0, 1, 2, ... In quantum mechanics, they obey different statistics of indistinguishable particles: same-kind fermions may exist only in anti-symmetric states, bosons—otherwise. We will discuss it later.

Quarks can exist only in colorless combinations (hadrons):

- 3 quarks of blue, green, red color: uud (proton), udd (neutron)
- quark-antiquark pairs in color-anticolor combinations (e.g., red-antired): $\bar{u}d$ (pion)

Experimentally most important hadrons (long-lived hadrons):

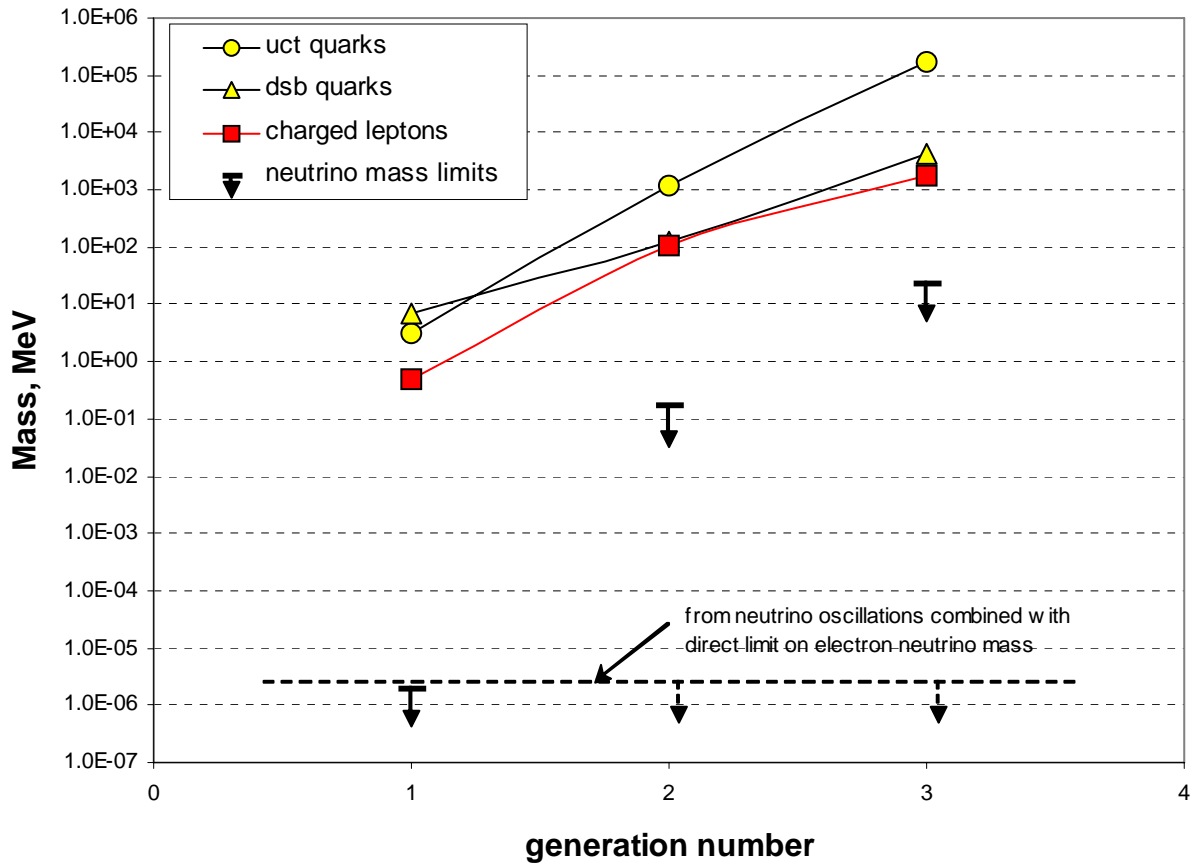
- proton uud
- neutron udd
- pion $\bar{u}d$
- kaon $\bar{s}d$

Anti-particles: For each particle in the table, there is anti-particle with the same mass/spin and opposite quantum numbers like charge (electric, color, etc.), magnetic moment, etc. For photon and Z, particle=anti-particle. Same can be true for neutrinos, but we do not yet know this...

W-boson is the only force via which particles can transform:

- e.g., $n \rightarrow p e \bar{\nu}$ (or at the more fundamental level: $d \rightarrow u e \bar{\nu}$)

Three Generations of Fermions: Pattern of Masses



Questions within Standard Model:

- **Origin of masses:** Is there a Higgs boson, after all? (Note that alternatives are possible...)
- **Neutrinos:** Are neutrinos Majorana- or Dirac-kind particles?
Also, what are neutrino masses, mixing angles and a CP-violating phase?
- **Quark-Gluon Plasma:** What are the properties of this state of matter?
This is sort of condensed matter physics for quarks and gluons...
- **Perturbative theory limitations:** How to deal with non-perturbative phenomena?
This is more a technical (“calculus”) question rather than physics...

What Standard Model does not explain:

- **Hierarchy problem:** Even if we find the Higgs boson in its simplest form, the Standard Model is likely to require new physics to avoid very bad divergences and/or fine tuning of parameters.
Is it Supersymmetry, Extra Dimensions?
- **Why does the Higgs boson, if discovered, couple so differently to various particles?**
- **Higgs field vacuum energy density is huge—problems for cosmology or extreme-fine tuning?**
- **Why are neutrinos so light?**
- **Why are there three generations of fermions?**
There appears to be a pattern in their masses...
What connects leptons to quarks?
What stands behind mixing of generations? There appears to be a pattern in the quark mixing matrix...
- **Why is there matter-antimatter asymmetry in the universe?** Baryon number conservation needs to be violated and we would need more CP-violating processes...
- **Is there new physics at the GUT scale?** Assuming existence of SUSY particles at ~ 1 TeV scale, all three coupling constants of the electromagnetic, weak and strong forces seem to converge to one value at 10^{16} GeV scale (well below Planck scale where we must incorporate quantum gravity). Is it a coincidence? Are there more gauge symmetries and corresponding forces?
- **What is non-baryonic dark matter?**
- **What is dark energy?**
- **Quantum field theory of gravity is missing...**
- **Why is the gravity so weak?**

20th Century Elementary Particle Physics Timeline

