Homework 1

18 + 7 (bonus) = 25 points

Problem 1 (1 point)

At what velocity, the relativistic expression for particle momentum $p=mv\gamma$ becomes 1% different form the non-relativistic formula $p=mv$.

Problem 2 (2 points)

Show that a photon of energy ε and scattered off an electron of mass m at rest ($\gamma + e^{-} \to \gamma + e^{-}$) would change its wavelength λ depending on the scattering angle θ according to the Compton scattering formula:

$$\Delta\lambda/\lambda = \varepsilon/m \cdot (1-\cos\theta)$$

Problem 3 (2 points)

A neutral particle decayed into proton p and negatively charged pion π^{-}. The momenta of charged particles were measured from the curvature of their tracks in magnetic field (in units of GeV):

	p_x	p_y	p_z
p	-0.488	-0.018	2.109
π^{-}	-0.255	-0.050	0.486

Find the mass and energy of the neutral particle. (2 points)

Problem 4 (5 + 3 bonus = 8 points)

A particle of mass M at rest decays into three particles A, B, and C of different masses m_A, m_B, m_C. Unlike in the case of a two-body decay, the energies of these three particles will not be the same in all decays, i.e., they can and will vary from decay to decay (of course, the total energy will be equal to M and total momentum—zero).

Case I: The particle A has its *minimum* allowed energy.
a) Draw the momentum vectors of all three particles in this case? (1 point)
b) What is this minimum energy for the particle A? (1 point)
c) What would the energies of the other two particles B and C be in this case? (2 points)

Case II: The particle A has its *maximum* allowed energy.
a) Draw the momentum vectors of all three particles in this case? (1 point)
b) What is this maximum energy for the particle A? (1 bonus point)
c) What would the energies of the other two particles B and C be in this case? (2 bonus points)

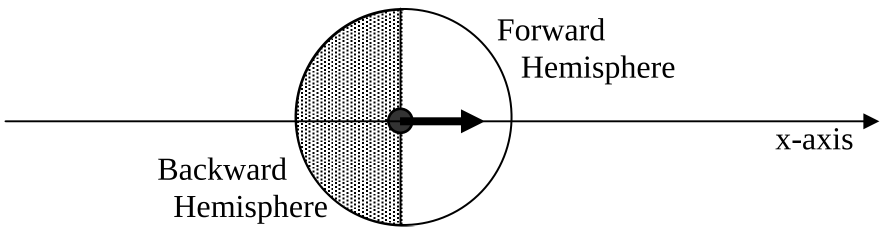
Hint: Recall a problem of a two-body decay that we considered in the class. Once you have figured out what needs to happen for a particle A to be at its minimum/maximum energy, the rest trivially follows from the two-body decay equations.

Problem 5 (4 + 2 bonus = 6 points)

Consider neutral pions π^0 of energy E=7 GeV decaying into two photons: $\pi^0 \to \gamma\gamma$.
1) What is the minimum opening angle between these two photons (in the lab coordinate system)? Draw directions of the photon momenta in lab and CM frames. (2 points)
2) What is the maximum angle (in the lab coordinate system)? Draw directions of the photon momenta in lab and CM frames. (2 points)
3) Guess which angle in the range from minimum to maximum is the most probable? Give your common sense reasoning or a formal prove, if you feel like doing it. (2 bonus points)

Problem 6 (2 points)

A charged pion with 4-vector momentum p=(E, P, 0, 0) flies along x-axis and decays in flight into a muon and neutrino. What must the pion energy E be so that the vector of the muon momentum would always point in the forward hemisphere?



Problem 7 (2+2 bonus = 4 points)

Some theorists speculate of a possibility of extra "curled" spatial dimensions. In a peculiar scenario when some of the extra dimensions are accessible to gravity only and the radius of "curvature" for these extra dimensions is relatively large, creation of tiny black holes might become possible at the Large Hadron Collider. (LHC is the future accelerator to be commissioned in 2007 and will provide proton-proton collisions with the center of mass energy of 14 TeV and rate of collisions of 10^9 Hz.) Theorists argue that such tiny black holes would evaporate well before having any remote chance of growing and swallowing the Earth… One may wonder if we should believe them on this point. Prove from the fact that the Sun exists for a few billion years, that, if such black holes, indeed, could be produced at the LHC, they would impose no danger whatsoever.

Hints:

a) *Estimate the energy E_0 needed for a proton colliding with a proton at rest to achieve the same center of mass energy as will be accessible at the LHC. (2 points)*

b) *Use the empiric formula for the flux of the high energy primary cosmic rays $dN/dE = a/E^3$ ($a \sim 10^{24}\ cm^{-2}s^{-1}sr^{-1}eV^2$) to find how many cosmic rays with an energy higher than the energy E_0 collide with the Sun every year. Sun radius is 7×10^8 m. (2 bonus points)*

Constants that you will need for these problems

Masses:	proton	938 MeV
	charged pion π^+	140 MeV
	neutral pion π^0	135 MeV
	muon	106 MeV
	neutrino	~ 0 MeV
	photon	= 0