Homework 2

15 + 8 (bonus) = 23 points

Problem 1 (4 points)

Find how thick a target of liquid Hydrogen should be to give 50-50 chances for scattering:

- 10 GeV protons (energy typical for cosmic rays)—2 points
- 10 MeV neutrinos (~ the highest energy solar neutrino)—2 points (express the answer in light years)

The total cross-section of pp scattering is σ_{pp}~40 mb in a very wide range of energies. The total cross-section of νp interaction is $\sigma_{\nu p}$~10^{-40} cm^2 for neutrinos with about 10 MeV energy.

Problem 2 (2 points)

The matrix element of weak interactions can be well approximated by a constant in a wide range of energies. How should the weak interaction cross section for process $\nu + n \rightarrow e^- + p$ depend on neutrino energy E in the lab frame where the proton target is at rest?

Problem 3 (2 points)

Typical width of excited states of proton and neutron is Γ~100 MeV.
What lifetime does it correspond to? (1 point)
How far can light travel during this time? Compare the answer with the proton radius of 1 fm. (1 point)

Problem 4 (3 points)

Imagine a particle X decaying into three different di-lepton channels with the following branching ratios:
$BR(X \rightarrow e^+e^-)=0.01$
$BR(X \rightarrow \mu^+\mu^-)=0.09$
$BR(X \rightarrow \tau^+\tau^-)=0.90$

An experiment detects many events with X-particle decays. One graduate student looks at a distribution of invariant masses of two muons in the $\mu^+\mu^-$-decay channel and finds that the distribution has a peak that she can fit with the following formula (*A* is some constant and mass *m* is in GeV):

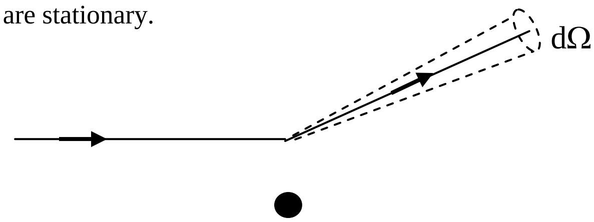
$$dN/dm = \frac{A}{(m-100)^2+9}$$

What is the lifetime of the X particle (express the answer in seconds)? (1 point)

Another student studies a distribution of invariant masses of two electrons. What is your prediction for the shape and normalization of this distribution? (2 points)

Problem 5 (4 points)

Non-relativistic <u>protons</u> with an initial momentum p scatter elastically off a gold thin-film target in particular direction (θ,φ) within a range of stereo angles $d\Omega$ with a cross section $d\sigma$. The cross section is dominated by interactions with the gold nuclei and the role of electrons can be neglected. Now consider a scattering of <u>α-particles</u> of the <u>same initial momentum</u> in the same stereo-angle angle direction and range. An α-particle is a nucleus of helium and has **<u>twice the charge</u>** and **<u>four times the mass</u>** of a proton. In both cases, one can assume that the gold nuclei, being so heavy, are stationary.



First, by what factor are the matrix elements m_{fi} for the two scatterings different? (2 points)
Second, taking into account all other factors, what is the α-particle cross section in terms of the proton cross section $d\sigma$? (2 points)

Problem 9 (8 bonus points)

Assume that protons are infinitely heavy. As you learned, if protons were positive point-like charges, the matrix element and cross section for elastic scattering of electrons on protons would be:

$$m_0(q) = \frac{e^2}{q^2} \qquad \text{and} \qquad \frac{d\sigma_0}{d\Omega} = \frac{1}{4\pi^2}\frac{e^4}{q^4}\frac{p^2}{v^2}$$

Find how these expressions would get modified, if protons were not point-like and had the spatial charge distributed in space as follows:

$$\rho(r) = Ae^{-r/r_0}.$$

Looking at the factors modifying the scattering cross section and assuming that r_0~1 fm, argue how large electron energy must be to allow one detecting the proton structure, if one could measure cross sections with δ~10% precision.

Hints:

1) *The normalization constant A must be such that the proton charge integrated over volume would be equal to e. Find this constant—2 points.*
2) *Calculate potential V(r) due to the charge distribution $\rho(r)$—2 points*
3) *Calculate matrix element M(q) for electron scattering off the potential V(r)—2 points*
4) *Define at what q the difference in cross sections would become larger that δ~10%. At what minimum electron energy such q would be possible?—2 points*

<u>Note</u>: These kind of deviation in electron scattering cross sections were seen in 1950s, which allowed to directly measure the proton size for the first time.