

## Natural Units

There are 4 primary SI units:

three kinematical (**meter, second, kilogram**) and one electrical (**Ampere**<sup>1</sup>)

It is common in the realm of the elementary particle physics to redefine units so that speed of light and Plank's constant become equal to one:  $c=1$  and  $\hbar=1$ . This imposes two constraints on the three kinematical units and, therefore, leaves us a free choice for one of the three kinematical units. The units of electrical charge, also, can be and are redefined (see below). Such system of units is often referred to as Natural Units (natural for the elementary particle physics, that is).

The kinematical unit of the choice is energy,  $E$ , and it is usually measured in eV (keV, MeV, GeV, TeV). Once we fixed  $c=1$  and  $\hbar=1$ , all other kinematical units can now be expressed in terms of units of energy. E.g., one can easily see:

$$E^2 = p^2 c^2 + m^2 c^4 \quad \text{with } c=1, \text{ units of } \mathbf{mass} \text{ and } \mathbf{momentum} \text{ are } E$$

$$\Psi = C e^{-i\left(\frac{Et - px}{\hbar}\right)} \quad \text{with } \hbar=1, \text{ units of } \mathbf{time} \text{ and } \mathbf{length} \text{ are } 1/E$$

(this can be also seen from  $E = h\nu$  and uncertainty principle  $\Delta x \Delta p \geq 1$ )

$$L_z = \hbar n \quad \text{with } \hbar=1, \mathbf{angular momentum} \text{ is dimensionless}$$

$$F = ma \quad \mathbf{force} \text{ units are } E^2$$

In SI system, units of current and charge can be *effectively* defined by choosing the value of  $\epsilon_0$  in the Coulomb's Law:

$$F = \frac{qq}{r^2} \frac{1}{4\pi\epsilon_0}, \text{ where } \epsilon_0 = 8.8 \times 10^{-12} \text{ C}^2 \text{m}^{-3} \text{kg}^{-1} \text{s}^2.$$

In the Natural Units, the units of charge are defined by choosing  $\epsilon_0=1$ . This automatically sets  $\mu_0=1$ , since  $c^2=1/(\epsilon_0\mu_0)$ . Since the units of force and distance in Natural Units are  $E^2$  and  $1/E$ , the electric charge turns out to be dimensionless in these units.

The value of the fine structure constant, being dimensionless, is the same in all units. In SI units it has the following form:  $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137}$ , where  $e$  is the elementary charge  $e=1.6 \times 10^{-19}$  C. In Natural

Units, the fine structure constant becomes  $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ . This clearly shows that an electric charge  $e$  has no dimensions in the Natural Units and now equals to 0.303.

**Table.** Units for major physical quantities in SI and Natural Units with conversion factors.

Quantity	Dimensions		Conversions
	SI Units	Natural Units	
mass	kg	E	1 GeV = $1.8 \times 10^{-27}$ kg
length	m	1/E	1 GeV <sup>-1</sup> = $0.197 \times 10^{-15}$ m
time	s	1/E	1 GeV <sup>-1</sup> = $6.58 \times 10^{-25}$ s
energy	kg·m <sup>2</sup> /s <sup>2</sup>	E	1 GeV = $1.6 \times 10^{-10}$ Joules
momentum	kg·m/s	E	1 GeV = $5.39 \times 10^{-19}$ kg·m/s
velocity	m/s	-	1 = $2.998 \times 10^8$ m/s (=c)
angular momentum	kg·m <sup>2</sup> /s	-	1 = $1.06 \times 10^{-34}$ J·s (=ħ)
cross-section	m <sup>2</sup>	1/E <sup>2</sup>	1 GeV <sup>-2</sup> = 0.389 mb = $0.389 \times 10^{-31}$ m <sup>2</sup>
force	kg·m/s <sup>2</sup>	E <sup>2</sup>	1 GeV <sup>2</sup> = $8.19 \times 10^5$ Newton
charge	C=A·s	-	1 = $5.28 \times 10^{-19}$ Coulomb; $e=0.303$ or $=1.6 \times 10^{-19}$ C

<sup>1</sup> Unit of charge, Coulomb, is Ampere · second.

## **Scales in Natural and SI units to remember**

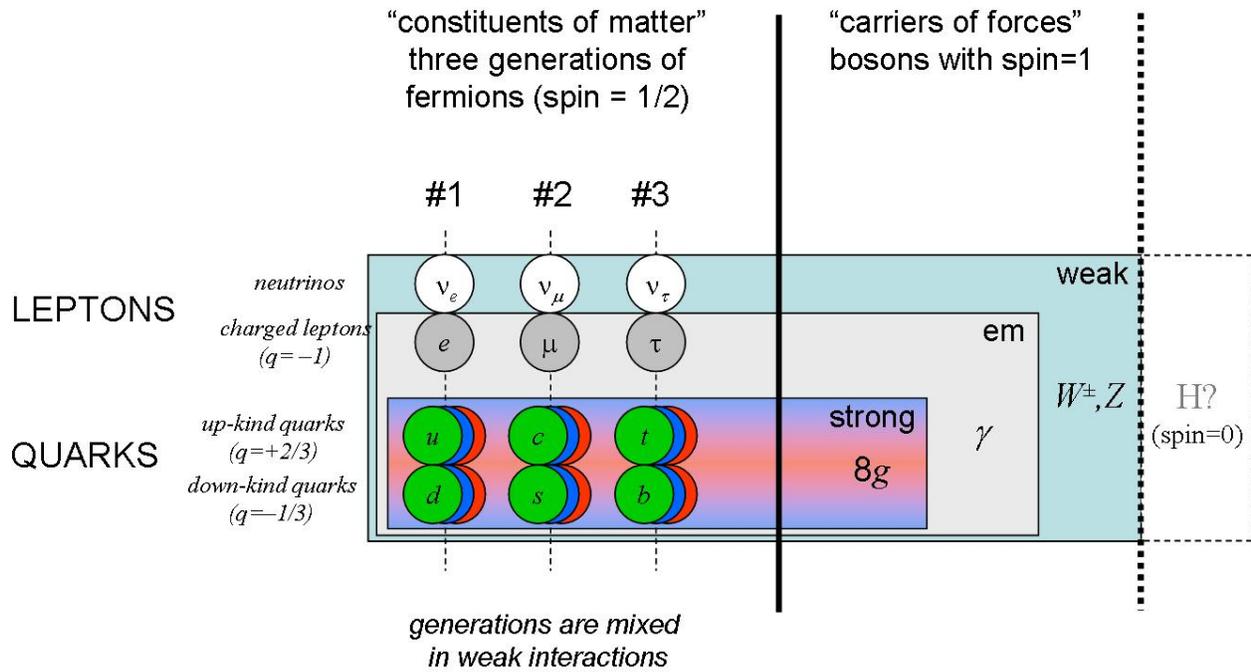
- mass of a proton/neutron  $\sim 1 \text{ GeV}$
- mass of an electron  $\sim 0.5 \text{ MeV}$
  
- proton/neutron radius  $\sim 1 \text{ fm} = 10^{-15} \text{ m} = 1 \text{ fermi} \sim 1/(200 \text{ MeV})$
- time for light to cross a proton  $\sim 10^{-23} \text{ s}$
  
- highest energy collider (Tevatron)  $\sim 2 \text{ TeV}$  center-of-mass energy
- distance scale probed at Tevatron  $\sim 1/(2 \text{ TeV}) \sim 10^{-19} \text{ m}$ ,  
(this comes from the uncertainty principle  $\delta x \cdot \delta p \sim 1$ )

### **Side note:**

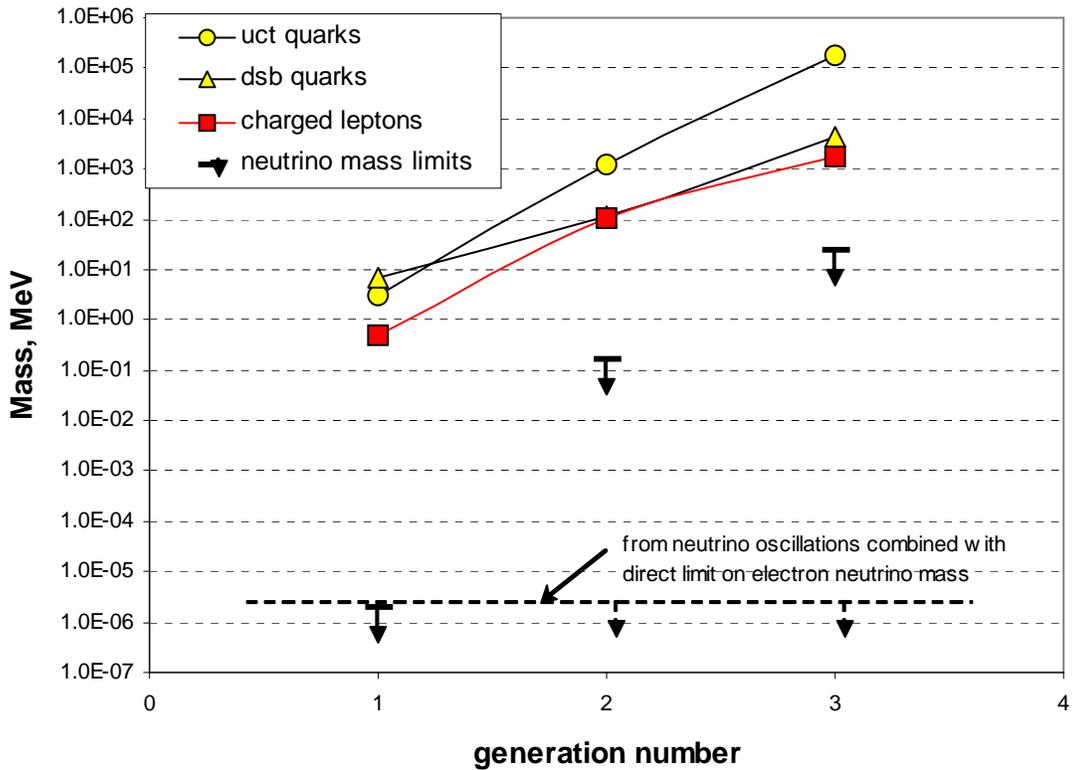
If you are still a little perplexed how it is possible to measure everything in terms of just one unit, think of astronomy, where the distance is basically measured in units of time:

- distance between the Earth and the Moon is about 1.3 s
- distance between the Earth and the Sun is about 8 min
- distance between the Sun and the nearest star is about 4 years, or light years.

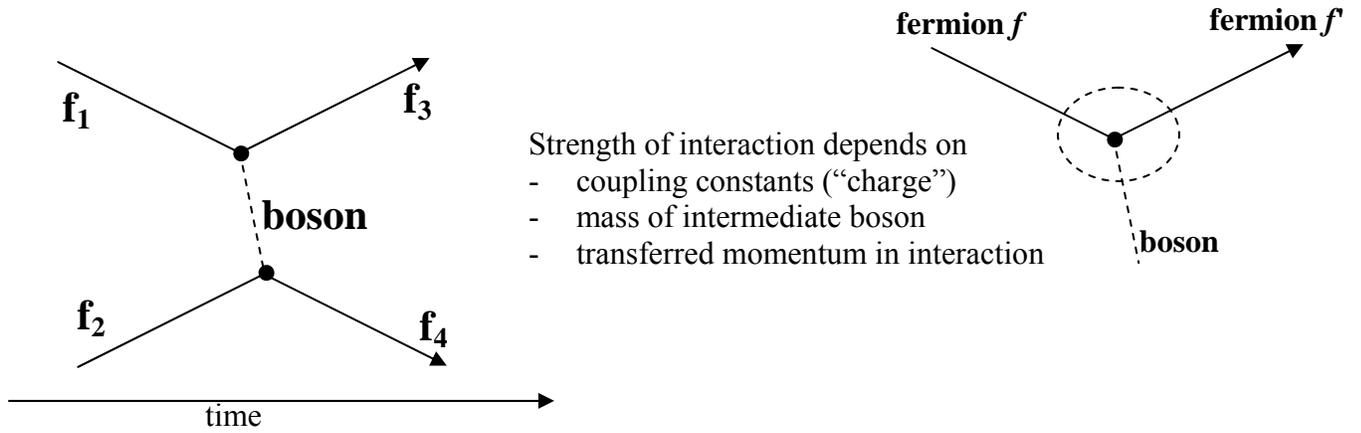
### Periodic Table of Particles/Forces in the Standard Model



### Three Generations of Fermions: Pattern of Masses



### Interactions via exchange with particles (Feynman diagrams)



<b>Intermediate bosons</b>	gluons	photons	Z/W	H
<b>Relative strength (quarks) at transferred momenta of ~1 GeV</b>	1	$\sim 10^{-2}$	$\sim 10^{-7}$	mass-dependent

Side Note: Boson Self-Coupling Vertices allowed in SM:

- **gluons:**            ggg;                            gggg
- **$\gamma, Z, W$ :**         $\gamma WW, ZWW$ ;             $\gamma\gamma WW, \gamma ZWW, ZZWW$
- **H, Z, W:**            HZZ, HWW, HHH;        HHHH, ZZHH, WWHH

**Particles and anti-particles:** For each particle, there is anti-particle with the same mass/spin and opposite quantum numbers like charge (electric, color, etc.), magnetic moment, etc. For photon, Z, and H, an anti-particle is the same as a particle. Same can be true for neutrinos, but we do not yet know this...

In general, fermions—particles with half-integral spin:  $\frac{1}{2}$ ,  $\frac{3}{2}$ , ... Bosons—particles with integral spin: 0, 1, 2, ... In quantum mechanics, they obey different statistics of indistinguishable particles: same-kind fermions may exist only in anti-symmetric states, bosons—otherwise. We will discuss it later.

**Fermions**, leptons and quarks with spin= $\frac{1}{2}$ , are conventionally called matter. Three columns represent 3 nearly identical generations; the masses being the only difference between them. This three-generation structure allows for not yet understood phenomena of mixing.

Each generation consists of:

- two leptons (charged lepton and an associated neutrino) and
- three pairs of quarks (colored blocks representing quarks with three different color charges)
- all fermions have masses

All particles have their own names:

- electron neutrino and electron, up- and down-quarks;
- muon neutrino and muon, charm- and strange-quarks;
- tau neutrino and tau-lepton, top- and bottom-quarks (*bottom* is also known as *beauty*);

**Quarks** have three kinds of strong force “charges”, which we call red, green, blue

Quarks form particles (**hadrons**), which must be color-neutral, or colorless

- 3 quarks of blue, green, red color
- quark-antiquark pairs in color-anticolor combinations (e.g., red-antired)
- Experimentally most important hadrons (long-lived hadrons): uud (proton), udd (neutron),  $\bar{u}d$  (pion),  $\bar{s}d$  (Kaon)

**Bosons:** There are 3 distinct kinds of bosons with spin=1 (mediators of forces):

- 8 gluons (spin=1) of different color-anticolor charges—responsible for the **strong force** (coupled to color charge, i.e. only quarks participate in interaction via exchange of gluons)
- 1 photon (spin=1)—responsible for **electromagnetic force** (coupled to electric charge, i.e. only charged particles participate in interaction via exchange of photons)
- Z- and W<sup>±</sup>-bosons (spin=1)—responsible for **weak force** (coupled to weak hypercharge and weak isospin; all fermions participate in interaction via exchange of Z and W). Unlike gluons and photons, Z and W are massive particles (91 and 80 GeV)

**Higgs boson** (spin=0)—*the only not yet discovered force/particle* (its mass >114 GeV), invented to allow for Z, W, and fermion masses in the *weak sector of SM*. Its coupling to particles is related to their masses: the larger the mass, the larger the force.

**W-boson is the only force via which particles can transform:**

- e.g.,  $n \rightarrow p e \bar{\nu}$  (or at the more fundamental level:  $d \rightarrow u e \bar{\nu}$ )

*Gravity is ignored (it is VERY weak and there is NOT a coherent quantum theory of gravity). Graviton should have zero mass and spin=2. The strength of gravitational interactions between elementary particles is imperishably smaller than even the weak force.*

## **Questions within Standard Model:**

- **Origin of masses:** Is there a Higgs boson, after all? (Note that alternatives are possible...)
- **Neutrinos:** Are neutrinos Majorana- or Dirac-kind particles?
  
- **Neutrinos:** What are neutrino masses, mixing angles and a CP-violating phase?  
This is a question of a measurement
- **Quark-Gluon Plasma:** What are the properties of this state of matter?  
This is sort of condensed matter physics for quarks and gluons...
- **Perturbative theory limitations:** Can we deal with non-perturbative phenomena? This is more a technical question (“calculus”) rather than fundamental physics

## What Standard Model does NOT explain:

- **Higgs:**
  - i. **Hierarchy problem:** Even if we find the Higgs boson in its simplest form, the Standard Model is likely to require new physics to avoid very bad divergences and/or fine tuning of parameters. Is it Supersymmetry (SUSY), Extra Dimensions?
  - ii. **Why does the Higgs boson, if discovered, couple so differently to various particles?**
  - iii. **Higgs field vacuum energy density is huge—problems for cosmology or extreme-fine tuning?**
- **Why are there three generations of fermions?**

There appears to be a pattern in their masses...  
What stands behind mixing of generations? There appears to be a pattern in the quark mixing matrix...  
What connects leptons to quarks?
- **Why are neutrinos so light?**
- **Why we do not see a CP-violation in strong force?**
- **Is there new physics at the GUT scale?** Assuming existence of SUSY particles at  $\sim 1$  TeV scale, all three coupling constants of the electromagnetic, weak and strong forces seem to converge to one value at  $10^{16}$  GeV scale (well below Plank scale where we must incorporate quantum gravity). Is it a coincidence? Are there more gauge symmetries and corresponding forces?
- **Why is there matter-antimatter asymmetry in the universe?** Baryon number conservation needs to be violated and we would need more CP-violating processes than we find with the SM.
- **What is non-baryonic dark matter?**
  - i. WIMPs? (SUSY could be an answer)
  - ii. Axions?
- **Gravity remains an untamed beast...**
  - i. Quantum field theory of gravity is missing... Is a string theory an answer?
  - ii. Why gravity is so weak?
  - iii. What is the dark energy, after all?

## 20<sup>th</sup> Century Elementary Particle Physics Timeline

