Interaction of particles with matter

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1. Particles and interactions

Four types of interactions: gravitational, weak, electro-magnetic, strong. The typical relative magnitudes of these forces: 10^{-39} : 10^{-7} : 10^{-2} : 1. The gravitational force plays no role in the high energy physics.

Particles. Consider only relatively long-lived particles so that they could travel some distance and had a chance to interact with matter before decaying. Note that due to γ-factor, the distance particles can travel before decaying will depend on their energy. We will use 100 GeV as a benchmark energy (particles of higher energy hardly ever produced even at Tevatron, the most powerful collider as of today).

Leptons:
- neutrinos (stable)
- electron (stable), muon (τ=2 μs, $c\tau$=500 m, $\gamma c\tau$=500 km), τ-lepton (τ=0.3 ps, $c\tau$=90 μm, $\gamma c\tau$=5 mm)

Hadrons: consider only ground states of quarks (excited states live ~10^{-21}-10^{-23} s)

Mesons—quark-antiquark pairs
- lightest mesons made of u,d-quarks, M~140 MeV
  - $\pi^\pm$ (τ=30 ns, $c\tau$=10 m, $\gamma c\tau$=6 km)
  - π^0 (τ=10^{-16} s, $c\tau$=30 nm, $\gamma c\tau$=20 μm)
- lightest mesons made of s- and u,d-quarks, M~500 MeV
  - $K^\pm$ (τ=10 ns, $c\tau$=4 m, $\gamma c\tau$=800 m)
  - K_L^0 (τ=50 ns, $c\tau$=20 m, $\gamma c\tau$= 4 km), K_S^0 (τ=0.1 ns, $c\tau$=3 cm, $\gamma c\tau$=6 m)
  - η (τ=10^{-18} s, …)
- lightest mesons made of c- and u,d,s-quarks, M~2 GeV
  - $D^\pm$ (τ=1 ps, $c\tau$=300 μm, $\gamma c\tau$=15 mm)
  - D^0 (τ=0.4 ps, $c\tau$=120 μm, $\gamma c\tau$= 6 mm)
  - $D_S^\pm$ (τ=0.5 ns, $c\tau$=150 μm, $\gamma c\tau$=8 mm)
  - J/ψ (τ=10^{-20} s, ….)
- lightest mesons made of b- and u,d,s,c-quarks, M~5 GeV
  - $B^\pm$, B^0, B_s^0 (τ=2 ps, $c\tau$=500 μm, $\gamma c\tau$=10 mm)
  - $B_c^\pm$ (τ=0.5 ps, $c\tau$=150 μm, $\gamma c\tau$= 3 mm)
  - Υ (….)
- no mesons are ever formed with t-quarks (they live much less than 10^{-23} s, hadron formation time)

Baryons—quark-quark-quark particles
- lightest baryons made of u,d-quarks, M~1 GeV
  - $p^\pm$ (stable)
  - n (τ=15 min, $c\tau$=3×10^8 km, $\gamma c\tau$=3×10^{10} km)
- lightest baryons made of s- and u,d-quarks, M~1 GeV
  - Λ (τ=0.3 ns, $c\tau$=8 cm, $\gamma c\tau$=800 m)
  - …

Carriers of the forces:
- photons (stable)
- gluons (do not occur as free particles)
- W/Z (live less than 10^{-25} s)

	Weak	**Electromagnetic**	**Strong**
relative strength	10^{-7}	10^{-2}	1
neutrinos	Yes	-	-
e, μ, τ	Yes	Yes	-
hadrons: charged	Yes	Yes	Yes
neutral	Yes	-	Yes
photons	-	Yes	-

2. Weak interactions (neutrinos)

The only particles for which this interaction is of any significance are neutrinos, as they do not participate in the other interactions. The force is extremely weak: the mean free pass of 1 MeV anti-neutrino in water is about 10^{20} cm or 50 light years! So detection of neutrino is actually detection of missing energy-momentum in the balance of the outgoing particles.

We never observe multiple neutrino interactions along its passage through matter. Whenever an extremely rare single interaction happens, other particles are produced (for example, $\overline{\nu}_e + p \to e^+ + n$) or knocked off from the media (for example, $\nu_e + e^-(atomic) \to \nu_e + e^-(scattered)$)—it is these particles that we will detect by making use of their interactions with matter via electromagnetic or strong force.

3. Electromagnetic interactions (charged particles)

3.1. Ionization energy losses for a charged particle passing through media

Bethe-Bloch formula for energy losses dE/dx (1930s)

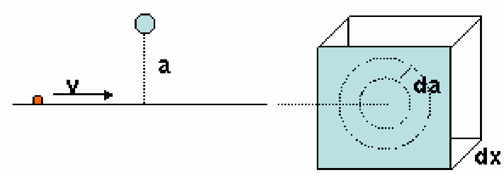
$$-\frac{dE}{dx} = 4\pi \frac{z^2\alpha^2}{\beta^2}\frac{Z\rho}{Am_N m_e}\left[\frac{1}{2}\ln\frac{2m_e\beta^2\gamma^2T_{\max}}{I^2} - \beta^2 - \frac{\delta}{2}\right],$$

where:

- m_e, m_N, α—universal constants: electron and nucleon masses; fine structure constant;
- z, β, γ—incoming particle parameters (charge in units of e, velocity β=v/c, gamma factor);
- Z, A, ρ, I—media properties: charge and atomic number of media atoms, density, average ionization potential for the media ($I \sim 16*Z^{0.9}$ eV for Z>1)
- δ—small correction due to media polarization (for gasses, it is negligibly small).

To understand the origin of the Bethe-Bloch formula, we will consider the following simplified model.

Momentum transfer from an incoming particle of charge *ze* to an atomic electron is:



$$dq_z = F_z dt = F_z\frac{dx}{v} = \frac{ze^2}{4\pi(x^2+a^2)}\frac{dx}{v}\frac{a}{\sqrt{x^2+a^2}},$$

where *a*—the shortest distance between particle's trajectory and an atomic electron; v—particle velocity, *ze*—particle charge, x—charged particle's coordinate along its trajectory. After integration from $-\infty$ to $+\infty$ in x we will have the total momentum transferred to an electron

$$q = 2\frac{z\alpha}{av}.$$

The energy transferred to electron (and, consequently, lost by the incoming particle) is then:

$$T = \frac{q^2}{2m_e} = 2\frac{z^2\alpha^2}{m_e v^2}\frac{1}{a^2}.$$

Summing up over all electrons in the media, the average energy losses experienced by the incoming particle after passing distance dx of the media will be:

$$dE = \int_{a_{\min}}^{a_{\max}} T\,2\pi a n_e\,da\,dx = 4\pi\frac{z^2\alpha^2}{m_e v^2}n_e dx\ln\left(\frac{a_{\max}}{a_{\min}}\right),$$

Note the two cutoffs in the integration a_{min} and a_{max}. At large distances, for which the calculated T becomes smaller than the ionization/excitation potential I, no energy transfer becomes possible—this imposes a cutoff a_{max}:

$$I = 2\frac{z^2\alpha^2}{m_e v^2}\frac{1}{a_{\max}^2}$$

The maximum energy an incoming particle can transfer to an electron is limited by the energy-momentum conservation laws. For an incoming particle of mass M, velocity v (and gamma-factor γ):

$$T_{\max} = \frac{2m\beta^2\gamma^2}{1+2\gamma\left(\frac{m}{M}\right)+\left(\frac{m}{M}\right)^2}$$

This can be accounted for introducing a cutoff a_{min}: $T_{\max} = 2\frac{z^2\alpha^2}{m_e v^2}\frac{1}{a_{\min}^2}.$

Therefore, the formula for energy losses that we are deriving can be re-written as follows (now the negative sign is also included to reflect that the energy is being lost):

$$-\frac{dE}{dx} = 4\pi \frac{z^2\alpha^2}{\beta^2}\frac{1}{m_e} n_e \left[\frac{1}{2}\ln\left(\frac{T_{\max}}{I}\right)\right]$$

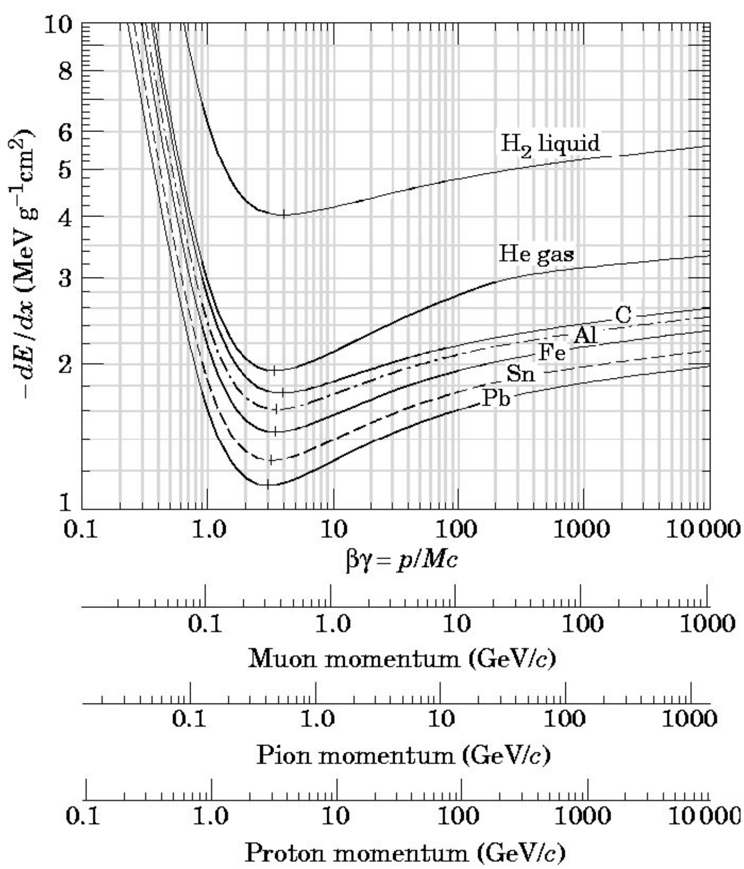
The number of electrons in the media of density ρ and made of elements (Z,A) is $n_e = Z\frac{\rho}{Am_N}$, from where:

$$-\frac{dE}{dx} = 4\pi \frac{z^2\alpha^2}{\beta^2}\frac{Z\rho}{Am_N m_e}\left[\frac{1}{2}\ln\left(\frac{T_{\max}}{I}\right)\right]$$

- Dependence on the charge *ze* of the incoming particle is as z^2.
- The energy looses decrease as $1/v^2$, where v is the velocity of the incoming particle. Slow moving particles would lose more energy and as their momenta increases (and the velocity saturates at the speed of light), one expects flattening out of dE/dx.
- If dE/dx is normalized on ρ, the dependence on media becomes very weak as Z/A~1/2 for most elements and =1 for hydrogen.

One can see that in comparison to the formula we have derived, the Bethe-Bloch formula has an extra factor of $\frac{2m_e\beta^2\gamma^2}{I}$ under the log, which actually leads to a slow rise of ionization losses with the particle momentum. This is due to accounting for the relativistic flattening of the electric filed of the incoming particle. As the result, the particle can ionize atoms at father and farther distance as its field becomes more and more squashed at larger momenta. This rise eventually flattens out due to polarization effects in the media. The plots below show the actual density-normalized dE/dx curves for a few materials.

- The typical value of energy losses at the minimum is about 2 MeV/(g/cm^2).
- One can see that the rise after the minimum is very slow and hardly exceeds 50% even at p~100 GeV. The particles with velocities corresponding to βγ>3 are usually called **minimum-ionizing particles** (**mip**).

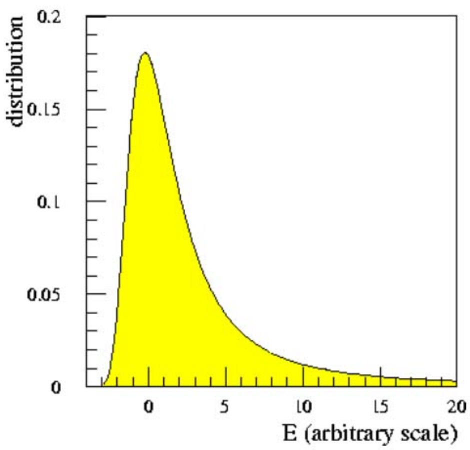


Landau fluctuations

The Bethe-Bloch formula gives the average energy losses for ionization and excitation. The fluctuations around the most probable value can be parameterized by the Landau distribution (these fluctuations are especially large for thin layers and gases):

$$L(\lambda) = \frac{1}{\sqrt{2\pi}} \exp\left\{ -\frac{1}{2}(\lambda + e^{-\lambda}) \right\},$$

where λ – is the deviation from most probable energy losses: $\lambda = \dfrac{\Delta E - \Delta E^W}{\xi}$ (ΔE – energy losses in a layer with thickness equal to x, ΔE^W – most probable energy losses, ξ - is a parameter characterizing the width of the distribution). Note that the dispersion of this distribution equals to infinity, independently of how small ξ is.



In gases,

- typical numbers of primary clusters ~ 30/cm
- typical total number of released electrons ~100/cm

In solids,
the numbers are ~1000 times larger (plainly due to their higher density).

Landau fluctuations (Landau tail) correspond to rear large energy transfers from the incident particle to atomic electrons. Such electrons are called **δ-electrons**. They typically can cause additional ionization, leading to a cluster of a few electrons.

If we know the average energy losses ΔE for a particle in media, the average number of released electrons can be estimated as ΔE/W, where W is the average energy spent per one released electron. W is somewhat larger than the ionization potential (I_0) because of some energy going into excitation of atoms and breaking of molecules. The number electrons appeared because of the interactions of the ionization particle with matter is $n_{primary}$, whereas n_{total} is the number of electrons appearing after interactions of initial electrons with matter.

Gas	density, ρ [g/cm^3]	I_0 [eV]	W [eV]	$n_{initial}$ [cm^{-1}]	n_{total} [cm^{-1}]
H_2	$8.99 \cdot 10^{-5}$	15.4	37	5.2	92
O_2	$1.43 \cdot 10^{-3}$	12.2	31	22	73
Ar	$1.78 \cdot 10^{-3}$	15.8	26	29	94
CO_2	$1.98 \cdot 10^{-3}$	13.7	33	34	91
CH_4	$7.17 \cdot 10^{-4}$	13.1	28	16	53
C_4H_{10}	$2.67 \cdot 10^{-3}$	10.8	23	46	195

Multiple scattering of charged particles

An interaction of a particle of charge *ze* with a nucleus of charge *Ze* charge is described by Rutherford formula (spins and not point-like structure of nuclei are ignored):

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\left(\frac{zZ\alpha}{\beta p}\right)^2 \frac{1}{\sin^4(\theta/2)}.$$

After passing through distance L and as a result of multiple scatterings on nuclei, the incident particle will experience some typical displacements and deflections. These will have approximately Gaussians distributions, whose averages are zero and sigmas are given as follows:

$$\theta_0 = \frac{14MeV}{\beta p} z\sqrt{\frac{L}{X_0}} \quad \text{and} \quad r_0 = \frac{1}{\sqrt{3}} L\theta_0$$

The Gaussian shape describes well the bulk of scatterings (98%), while the tails exhibit $\sim 1/\sin^4(\theta/2)$ dependence coming from the Rutherford formula.

Dependence of θ_0 on particle's charge, velocity and momentum can be easily traced to the Rutherford formula as well. Also, it is clear that the dependence on the material thickness should be like square root since the overall scattering is a accumulation of independent scatterings.

X_0 is the characteristic of media and is called the radiation length. The dependence of X_0 on material parameters is intuitively clear: it must be inversely proportional to $n_A=\rho/A$ (density of nuclei, scattering centers, in media) and inversely proportional to $Z^2+Z=Z(Z+1)$ (scattering on nuclei $\sim Z^2e^2$ plus scattering on electrons $\sim e^2$, but whose number Z times larger):

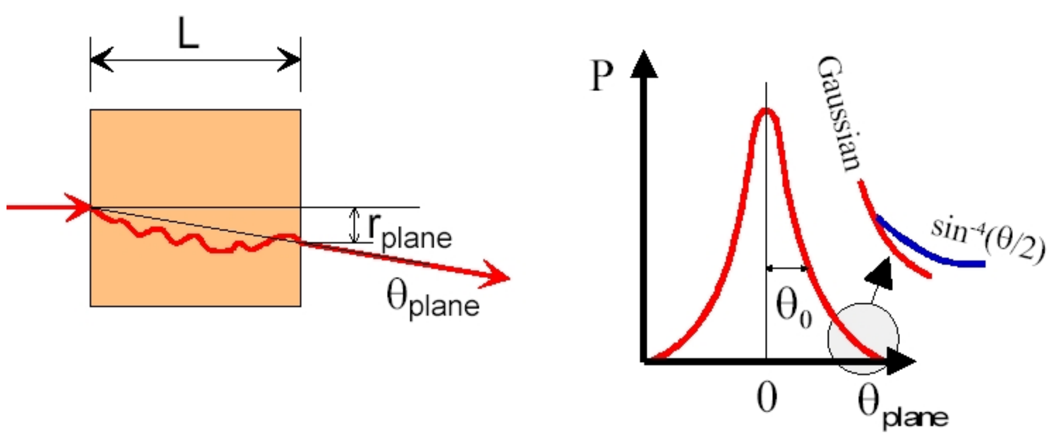
$$X_0 = \frac{m_e^2}{4Z(Z+1)\alpha^3} \frac{1}{n_A} \frac{1}{\ln(183/Z^{1/3})} + \ldots$$

The following formula is good for quick estimates (log term is different, which effectively accounts for small extra factor omitted in the formula above):

$$X_0 \approx \frac{\left(716g/cm^2\right)}{Z(Z+1)\ln(287/\sqrt{Z})} \frac{A}{\rho}$$

- Typical values of X_0 are 0.6 cm (lead), 1.8 cm (iron), 36 cm (water), 30 m (air).

- After traversing 1 m of iron, a muon of 10-GeV energy will be
  - typically deflected by ~0.6° and
  - typically displaced by ~6 mm
- Same numbers for air will be about sqrt(3000/1.8)=40 times smaller…



Scintillation

A charged particle traversing matter leaves behind it a wake of excited molecules. Certain types of molecules, will release a small fraction of this energy in the form of optical or close UV photons, for which the media may be quite transparent with attenuation lengths reaching as much as a few meters. Amount of energy carried away by scintillation light is typically 1% or less of dE/dx. This light may be used for detecting the fact of a particle traversing the media.

Scintillating materials are classified in inorganic and organic.

Inorganic

- high-Z, very high density (good for γ and *e* detection and energy measurement)
- high light yiled
- relatively slow

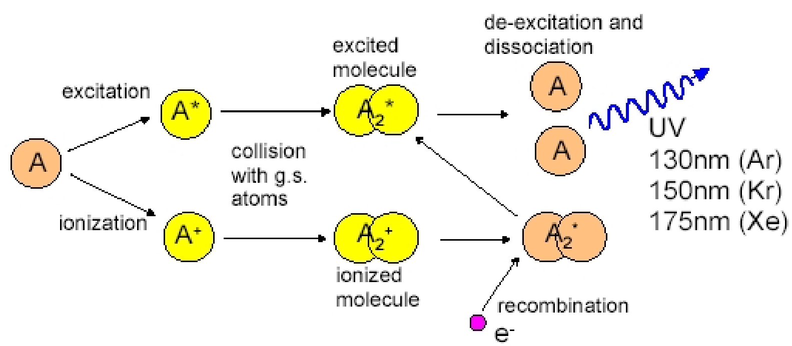
Classical Examples	decay time (ns)	λ (nm)	γ per MeV dE/dx	X_0 (cm)
NaI (very hygroscopic!)	60	250	4×10^4	2.6
CsI	1000	570	1×10^4	1.9
$Bi_4Ge_3O_{12}$ (BGO)	300	480	3×10^3	1.1
$PbWO_4$	10	430	4×10^2	0.9

Organic (plastics, liquid)

- low cost and ease of fabricating various shapes
- fast
- relatively smaller light yield
- Hydrogen reach (good neutron detection)

Classical Examples	decay time (ns)	λ (nm)	γ per MeV dE/dx	X_0 (cm)
NE-104	2	400	1×10^4	40
BC-408	2	425	1×10^4	40

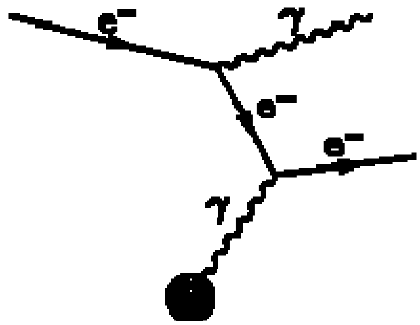
Below is an example of scintillation mechanism in noble gases (note that the original excitations have much larger energy than the energy of the scintillation light).



3.2 Bremsstrahlung (radiation losses or “breaking radiation”)

Bethe and Heitler, 1934

Relativistic charged particles, as they propagate through matter and wiggle due to multiple scattering on nuclei, experience accelerations and, therefore, must be radiating electromagnetic waves—emission of such photons is called "breaking" radiation, or bremsstrahlung.



The energy losses turn out to be proportional to the incoming particle energy, while the proportionality coefficient $1/X_0$ depends on the composition of the material as well as the incoming particle mass and charge.

$$-\frac{dE}{dx}=\frac{E}{X_0},$$

where X_0 is the radiation length (discussed in section on multiple scattering):

$$\frac{1}{X_0}=\frac{4Z(Z+1)\alpha^3}{m_e^2}n_A\ln\frac{183}{Z^{1/3}}$$

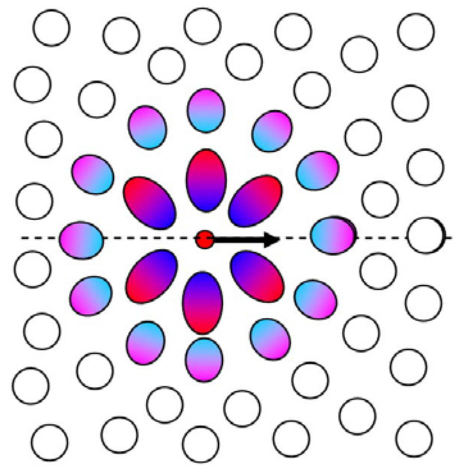
The energy at which bremsstrahlung radiation becomes equal to ionization losses is called critical energy E_c. Below E_c, the ionization losses dominate; above E_c, the main source of energy losses is bremsstrahlung. For media atoms with $Z\geq 13$ the critical energy values for incident electrons can be estimated as follows:

$$E_C=\frac{550MeV}{Z}.$$

The critical energy for electrons in iron (Z=26) is ~20 MeV, 7 MeV in lead (Z=82).

The amount of multiple scattering depends on particle's momentum and does not depend on particle's mass. The amount of radiation, however, depends quadraticly on the acceleration and, therefore, is proportional to $\sim 1/m^2$. For example, breaking radiation for muons, the lightest charged particle after an electron, will be 40,000 times smaller than for electrons of the same momentum. The critical energy for muons will be ~1 TeV.

3.3 Cherenkov radiation

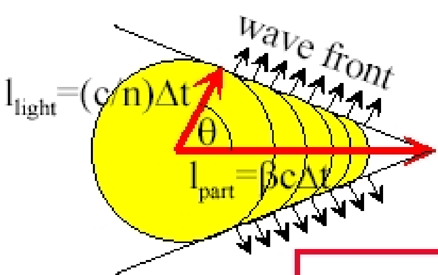


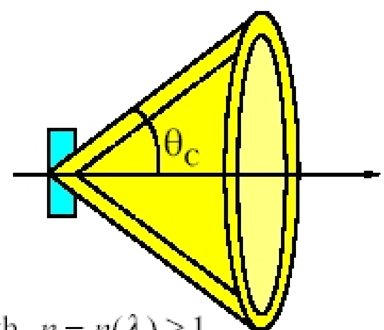
As a charged particle traverse matter, it produces a wake of polarized molecules along its path. As the molecules get polarized and then de-polarized, they emit radiation in all directions.

If the particle moves with a speed v exceeding speed of light in the media c/n (n is the refraction index for the media), in a certain direction the radiation becomes *coherent* and results in significant amount of light produced. The direction in which waves would add up coherently can be easily constructed using Huygens technique of circular fronts.

This radiation is called Cherenkov (often spelled Čerenkov) after Cherenkov who has studied it in mid-1930s and showed that it was not due to luminicense or any other known radiation mechanism. The first observation is actually attributed to Mallet in 1926. The theory behind the Cherenkov radiation was put together by Frank and Tamm in 1937.

$$\beta \geq \beta_{thr} = \frac{1}{n} \quad n: \text{refractive index}$$





$$\boxed{\cos\theta_C = \frac{1}{n\beta}} \quad \text{with} \quad n = n(\lambda) \geq 1$$

The main formulas related to Cherenkov radiation are the ones for its direction and intensity:

- $\cos\theta_C = 1/nv$ - angle between a particle direction and photon emition direction
- $\frac{dN}{dxd\varepsilon} = 2\pi\alpha z^2\left(1-\frac{1}{n^2v^2}\right)$ - number of phtons emmited per unit of energy (flat spectrum)

One can see that there is a threshold minimul velocity v=1/n, below which there is no radiation. As the particle surpasses this threshold velocity, it starts emitting small amount of light in the forward direction (θ~0). As its velocity approaches speed of light, the intensity reaches its maximum and opens up to the maximum angle θ_{max}=acos(1/n). Amount of energy emitted is about 10^{-4} of dE/dx losses.

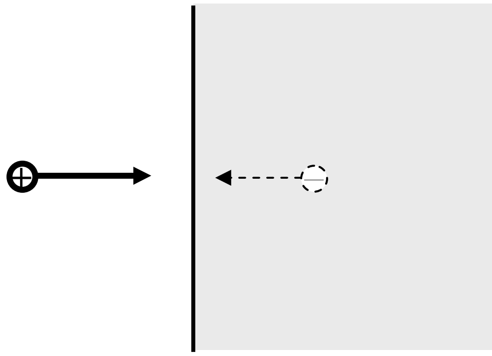
A few reference numbers for some common matrerials:

Media	n	θ_{max}	dN/dxdε ($cm^{-1}eV^{-1}$)	visible dN/dx (cm^{-1})	$\gamma_{threshold}$
Air	1.000283	1.36°	0.208	0.26	42
Isobutane	1.00127	2.89°	0.941	1.2	20
Water	1.33	41.2°	161	213	1.5
Quartz	1.46	46.7°	196	261	1.4

1958 **Cherenkov, Frank, Tamm** awarded Nobel Prize for discovery and explanation of the effect.

3.4 Transition radiation

This radiation was predicted by Ginsburg and Frank in 1946.
The correct relativistic treatment is due to Garibian, 1958.



The field of charged particle in vicinity of a boundary with dielectric material can be calculated using charge image technique. The particle and its image form a dipole. If the particle moves towards or away from the boundary, the dipole moment d will be changing with tame and, therefore, one should expect the characteristic dipole radiation to occur.

Average radiated energy per boundary: $W = \frac{1}{3}\alpha\omega_p\gamma$,

where $\omega_p = \sqrt{\frac{N_e e^2}{\varepsilon_0 m_e}}$ (~20 eV for plastics).

- The typical energy of photons is $\varepsilon = (\omega_p / 4) \cdot \gamma$ (therefore, fast moving particles produce X-rays)
- The average number of emitted X-ray photons per boundary N=α=1/137.
- Note that the intensity of transition radiation (energy of X-rays) increases lineraly with particles energy, or more accurately with γ.
- The angular distribution of transition radiation is peaked forward with a sharp maximum at θ=1/γ.
- Radiation from vacuum-media and media-vacuum boundaries has different phases and therefore a minimum thickness of the film is required to prevent coherent concelation. The minimum thickness of materials is $\Delta \sim (c/\omega_p)\gamma^2$: ~20 μm for CH_2 plastic polimers and ~1 mm for air gaps.

4. Electromagnetic interactions (photons)

There are three very distinct processes of photon interacting with media:

- $\gamma + atom \rightarrow atom^+ + e^-$ (photoelectric effect; dominant for $\varepsilon_\gamma < 0.1$ MeV)
- $\gamma + e^- \rightarrow \gamma + e^-$ (Compton effect; dominant for 0.1 MeV < ε_γ < 10 MeV)
- $\gamma + nuclear \rightarrow e^+ + e^- + nuclear$ (Pair production; dominant for ε_γ > 10 MeV)

The photon beam intensity in a media falls exponentially: $I = I_0 e^{-x/\lambda}$,

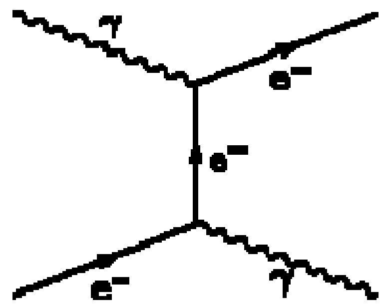
where λ is a free mean path, $\lambda = \dfrac{1}{n_A \sigma_{total}} = \dfrac{1}{n_A \sum_i \sigma_i}$.

4.1 Photoelectric process, or photo-effect

This is the process of photo absorption leading to ionization of an atom. If photon energy is sufficient, an electron from the most inner atomic shells (K-shell) will be predominantly knocked off. In this case, an electron from a higher energy shell can fall and emit a characteristic frequency light. (Auger-electrons can be discussed here as well). Approximately, the photo-effect cross section away from the characteristic energy peaks depends on the element properties and photon energy as

$$\sigma_{ph} \propto \frac{Z^5 \alpha^4}{\varepsilon^3}.$$

4.2 Compton scattering



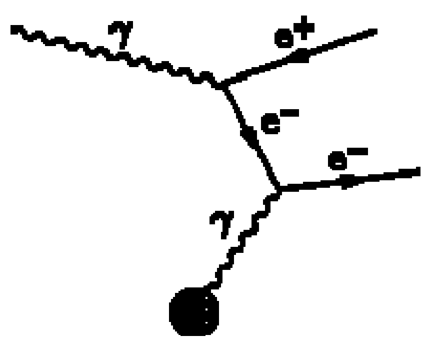
This is the process of scattering of photons on atomic electrons. Approximately, the Compton cross section has the following dependence on media properties and photon energy as:

$$\sigma_C \propto Z\alpha^2 \frac{\ln \varepsilon}{\varepsilon}$$

The t-channel diagram is shown on the right. There is also a similar s-channel diagram. One can see from these diagrams that the matrix element will be proportional to e^2. Therefore, the atomic cross section will be proportional to $Z \cdot (e^2)^2$, where the extra factor Z accounts for the number of electrons in an atom.

The energy spectrum of scattered photons and knocked off electron is approximately flat from ε=0 to $\varepsilon = \varepsilon_\gamma$.

4.3 Pair Production



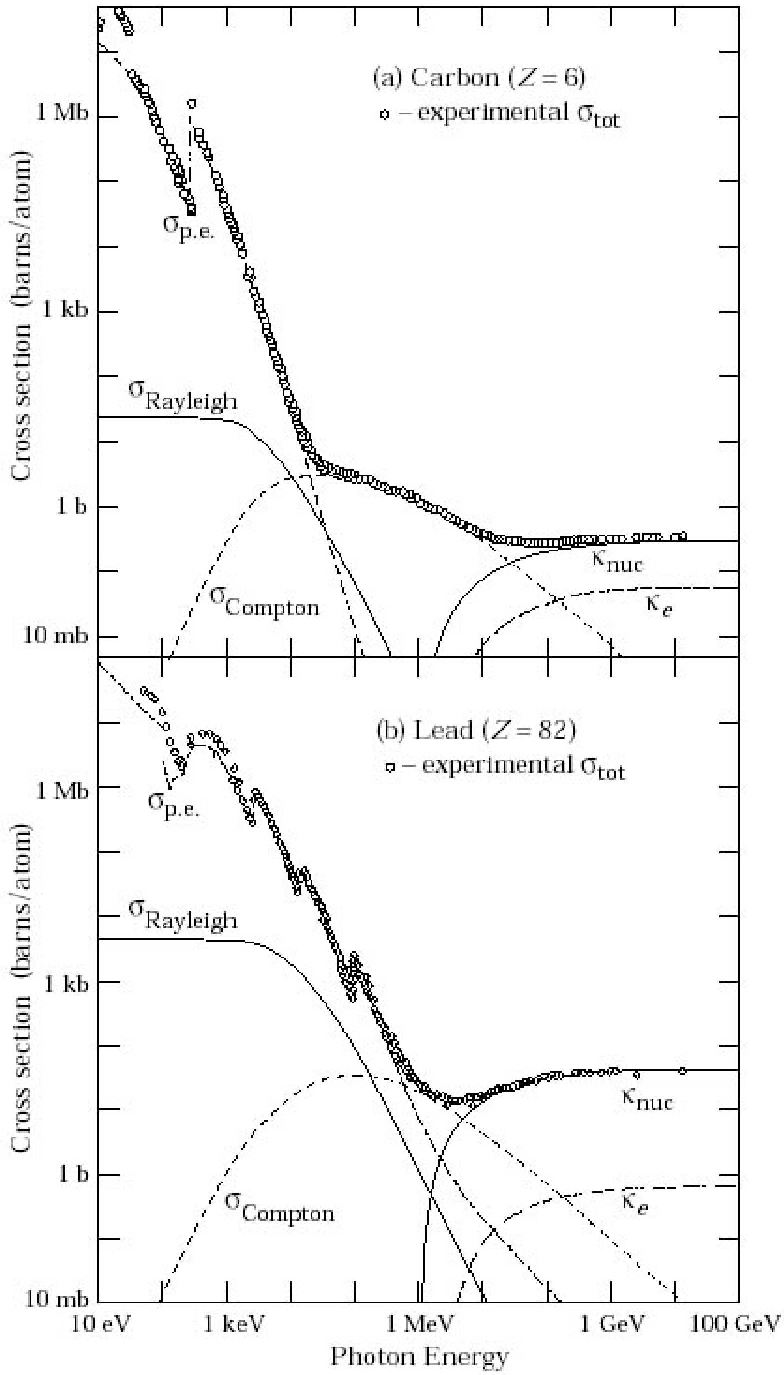
This is the process e^+e^- -pair production by photons in the nuclear field.
This process has an energy threshold and only possible for $\varepsilon_\gamma > 2m_e = 1$ MeV. For photon energies $\varepsilon_\gamma > (70 \text{ MeV}) / Z^{1/3}$, the cross section reaches the plateau level and become practically energy independent (the dependence on α and Z is obvious):

$$\sigma_{pair} = \frac{7}{9}\left(\frac{4\alpha^3 Z^2}{m_e^2} \ln \frac{183}{Z^{1/3}}\right).$$

Note the interesting relationship that follows from this equation:

$$\lambda_{pair} \sim (9/7) \cdot X_0$$

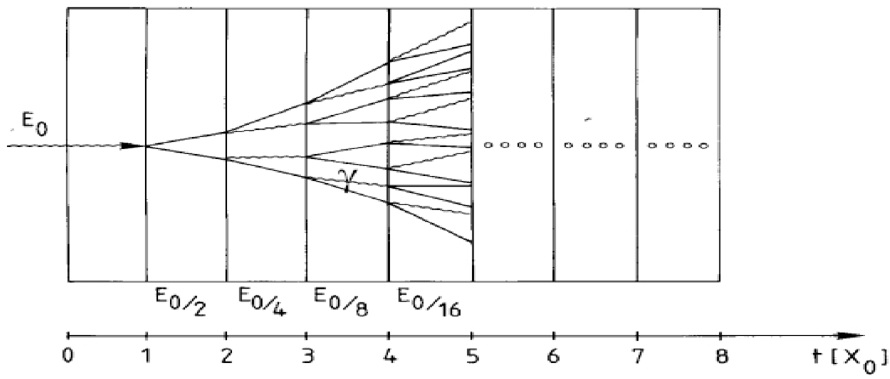
From the obvious electron-positron symmetry in this process, the average energies of electrons and positrons must be equal to $\varepsilon_\gamma/2$.



Total photon cross section σ_{tot} in lead, as a function of energy:

$\sigma_{p.e.}$, atomic photo-effect (electron ejection, photon absorption);
$\sigma_{Rayleigh}$, coherent scattering (Rayleigh scattering—atom neither ionized nor excited);
$\sigma_{Compton}$, incoherent scattering (Compton scattering off an electron);
κ_{nuc}, pair production, nuclear field;
κ_e, pair production, electron field.

5. Electromagnetic showers (electrons and photons)



The picture above illustrate a simplified model of an electromagnetic shower. Only bremsstrahlung and pair production processes are considered. For simplicity, we will assume $\lambda_{pair} = X_0$.

One can see that the number of particles grows with the distance as: $N(t) = 2^t$ and energy per particle falls at each step as $E(t) = E_0 2^{-t}$.

Process continues until $E(t) < E_c$ --after this point electrons will be losing their energy predominantly via ionization losses and photons will be re-scattered a few times (Compton) and finally absorbed (photo-effect). The process of particle multiplication stops when $\frac{E_0}{2^{t_{\max}}} = E_c$, from where $t_{\max} = \frac{\ln E_0 / E_c}{\ln 2}$.

Note the shower size in length grows with energy only logarithmically.

Total number of particles produced in the shower is *proportional to the incident particle energy*:

$$N_{total} = \sum_{t=0}^{t_{\max}} 2^t = \frac{a_n q - a_0}{q-1} = \frac{2^{(t_{\max}+1)} - 1}{2-1} \approx 2 \cdot 2^{t_{\max}} = 2\frac{E_0}{E_c}.$$

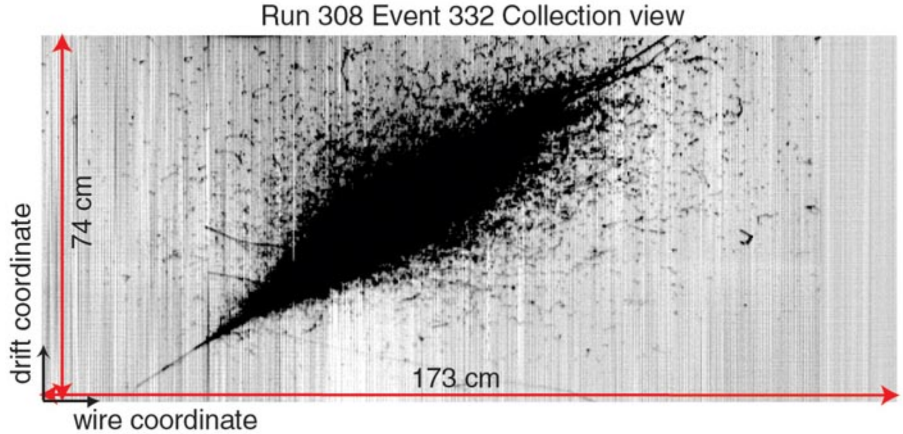
95% of the shower core is contained in a cylinder of radius $R_{95\%} = 2R_M$ and length $L_{95\%} \sim t_{\max} + 10$, where

$R_M = \frac{21MeV}{E_c} X_0 \left[g/cm^2 \right]$ is Molier radius.

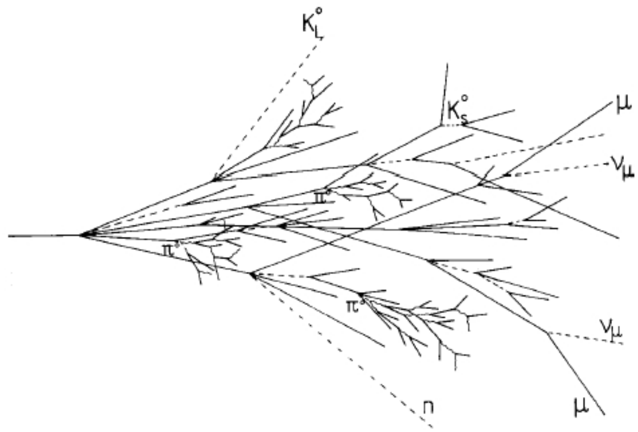
For example, for lead and 100-GeV electrons, $R_{95\%} \sim 4$ cm and $L_{95\%} \sim 16$ cm.

Picture below shows an electromagnetic shower recorded by the ICARUS experiment in their Liquid Argon Time Projection Chamber.

Run 308 Event 332 Collection view



6. Strong interactions: Hadronic shower (hadrons)



High energy collisions of hadrons typically result in prolific production of pions, the lightest particles made of quarks. Strong interactions of a kind $pion + p \to k\ pions + p$ will, therefore, naturally lead to a hadronic cascade similar in many respects to an electromagnetic shower produced by electrons and photons. The main differences will be in a) mean free path between collisions;
b) number of particles produced per collision;
c) minimum energy after which the shower development terminates.

From point of view of strong interactions, protons and neutrons approximately can be viewed as non-transparent spheres of radius r~1 fm. Therefore, $\pi+p$ cross section is $\sigma_0 \sim \pi r^2 \sim 30$ mb. A nucleus of A protons and neutrons, in its turn, can be viewed as a bag of tightly packed nucleons and its cross section, therefore, can be estimated as $\sigma = \sigma_0 \cdot A^{2/3}$.

Mean free pass for a hadron in media of atoms (Z,A) is then $\lambda = \dfrac{1}{n\sigma}$, where n is atomic density: $n = \rho/(Am_N)$.

Thus, one can estimate mean free pass between interactions $\lambda = \dfrac{1}{n\sigma} = \dfrac{Am_N}{\rho\sigma} = \dfrac{A^{1/3} m_N}{\rho\sigma_0}$.

For iron, this gives λ~19 cm. The actual number is close and somewhat smaller (17 cm)—this is because we ignored interactions that would lead to nucleus excitations.

Number of particles produced per one collision k typically will be somewhat larger than 2 as in the case electromagnetic shower ($e \to e + \gamma$ or $\gamma \to e^+ + e^-$).

The hadron cascade will stop when the energy of products reaches ~200 MeV, after which no more pions can be made (pion mass ~140 MeV). The total number of particles in the shower and the point at which it reaches its maximum can be estimated the same way as was done for em-shower (albeit small modification: 2 → k):

$L_{\max} = \lambda \dfrac{\ln E_0 / E_{\min}}{\ln k}$, which for 100 GeV hadrons gives $L_{\max} \sim 6\lambda$. The transverse shower size ~λ.

There are two more important differences from the em showers that were not mentioned above:

d) The number of particle species that can in principal be produced is large
e) The most prolifically produced particles are pions: π^+, π^-, π^0. However, π^0 is a short lived particle that decays into two photons. Therefore, all π^0s, once produced, decay to photons, which start dense and relatively short electromagnetic showers.

Therefore, although the number of charged tracks produced in a hadronic cascade is proportional to the energy of an incoming particle, the fluctuations are typically much larger than those in the electromagnetic shower initiated by an electron or photon.