

“Discovering” QED, QCD, and Electroweak theories

This note is intended to highlight the main ideas and discoveries related to the development of the theory of electromagnetic, strong, and weak forces.

Status as of 1930s and early 1940s

Electromagnetic Force:

Dirac equation for spin-1/2 particles combined with electromagnetic field equations (spin-1 massless field) led to formulation of Quantum ElectroDynamics, or QED. Standard perturbation techniques lead to spectacular successes, e.g., in describing atomic spectroscopy.

However, 2nd order loop-corrections were found to be infinite which plagued the further development...

Weak Force:

Enrico Fermi formulated in 1933 a 4-fermion model of weak interactions with a plain constant matrix element $M=G_F \sim 10^{-5} \text{ GeV}^{-2}$. This theory beautifully described at the time all known weak force mediated phenomena, e.g., neutron decay.

But it leads to rising cross-section with energy (see earlier lectures): $\sigma \sim G^2/(4\pi) \cdot s \cdot (\text{spin factors})$. This can also be easily seen from dimensional analysis $\sigma [\text{GeV}^{-2}] \sim M^2 s [\text{GeV}^4 \cdot \text{GeV}^2]$. The total cross-section for scattering with a particular angular momentum cannot exceed the unitarity limit of $\sigma_0 = 4\pi(J+1)/s$. The Fermi model cross-sections would exceed the limit at a few hundreds GeV (accurate calculations give $\sqrt{s} \sim 300 \text{ GeV}$).

In addition, theoretically, the Fermi model works only in the first order... 2nd-order divergences are actually much worse than in QED...

The problem could be overcome by assuming that there is a force mediator, a charged heavy particle of mass m and coupling g such that $M=g^2/(m^2+q^2)$ had a low energy limit $g^2/m^2 = G_F$.

Strong Force:

The best take on that was made by Yukawa in 1935. He assumed the existence of $\sim 100 \text{ MeV}$ particle that would be the mediator of the force. But no real quantitative theory existed...

Quantum Electrodynamics (QED)

1947-8 **Tomanaga, Schwinger, and Feynman** independently find a solution for the loop-divergences. It is called renormalization: the diverging integrals can be separated and uniquely included in the coupling constants and particle masses. Thus one can redefine the charge and mass that we observe in the experiment and that are finite as a combination of "bare", unphysical, infinite charges and masses and magically canceling them divergent integrals. The procedure may sound ugly (it is often referred to as *sweeping the problem under the rug*), but it actually works to astonishing precision. This moment can be considered the birth of the coherent theory of QED. We will discuss this later.

QED: Carrier of the force: massless spin-1 neutral photons
 Particles subject to the force: any particle carrying electric charge (particles are allowed to have masses)
 Electromagnetic interactions: any interaction involving photons

Tomanaga, Schwinger, and Feynman shared the Nobel Prize in 1965 "for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles"



Important note:

Local gauge invariance:

Require that a Lagrangian is invariant under gauge transformation: $\psi(x) \rightarrow e^{iq\theta(x)}\psi(x)$, where q is an electric charge and $\theta(x)$ is an arbitrary function of coordinates. When there are more than one kind of "charge", ψ should be treated as a vector (components become different "charges") and θ becomes a matrix.

Local gauge invariance typically results in renormalizable theories.

The local gauge invariance can be satisfied only if the carriers of the force are massless.

Caveats:

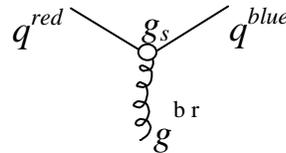
Strictly speaking, in an Abelian theory where there is only one kind of charge (QED is an example of such a theory), one can write Lagrangian which is not gauge invariant, but would nevertheless result in renormalizable theory; in particular, "photons" can be massive without breaking renormalization.

Also, local gauge invariance by itself is not an absolutely sufficient condition for renormalizability.

Quantum Chromodynamics (QCD)

- 1954 **Young, Mills** introduced a theoretical framework for massless, spin-1, self-interacting particles. This will become a starting point for the Strong force and ElectroWeak theories.
- 1963 **Gell-Mann** and **Zweig** proposed a model of quarks
- 1963 **O. W. Greenberg** suggested that quarks occur with three different color charges: *red, green, blue*.
- 1969 **J. D. Bjorken** argues on phenomenological grounds that the results of the deep inelastic scattering imply that a proton, when probed with sufficiently large momenta, looks like a bunch of essentially free unbound partons (so-called Bjorken scaling). This was extremely amazing as no one could think of how the short distance strong force could be at the same time very weak at sub-nucleon scales.
- 1971 **G. 't Hooft** proved that the theory of massless Young-Mills fields was renormalizable.

All one had to do is to arrange quarks in three-color triplets and allow transitions between them by means of Young-Mills fields. This automatically gave an octet (8) of color-anticolor charged massless spin-1 carries of the force that we call gluons¹.

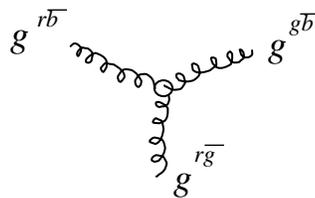


- 1973 **Gross+Wilczek, Politzer** (two independent papers) obtain the "asymptotic freedom" property of interacting Young-Mills field theories. This is the resolution of the apparent Bjorken scaling paradox!
- 1973 **Weinberg, Fritzh+GellMann+Leutwyler, Gross+Wilczek** (three independent papers) write down the final QCD Lagrangian. The Lagrangian can be easily built by imposing the local gauge invariance with three color charges (cf one kind of charge in QED).

QCD: Carrier of the force: massless spin-1 self-interacting gluons (8 of them)
 Particles subject to the force:

- quarks carrying color charge (particles are allowed to have masses)
- gluons themselves, as they carry color charges

Strong interactions: any interaction involving gluons



The property of asymptotic freedom, when turned around, actually implies that the strength of quark interactions becomes larger and larger as the characteristic momentum transfer decrease. I.e., the strong force keeps increasing as the distance separating quarks gets larger. This is what probably causes quarks to be forever confined in hadrons—the property of confinement.

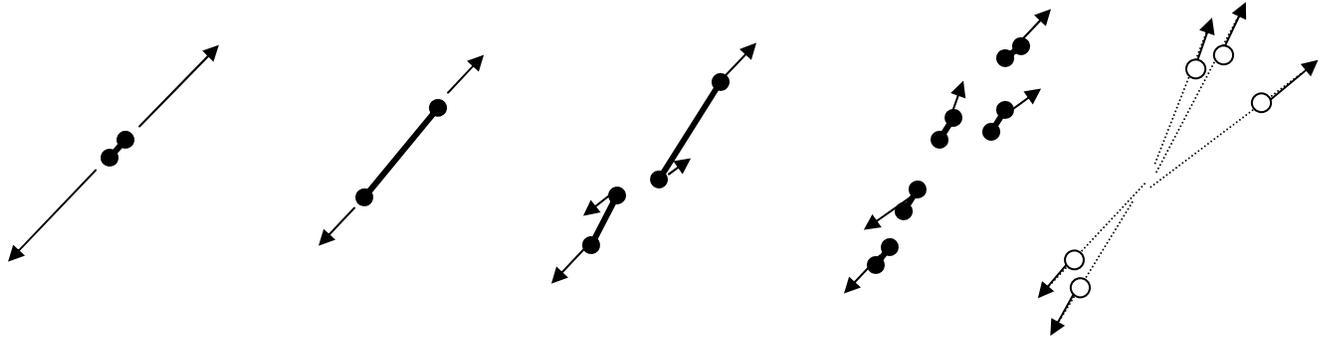
- 2004 **Gross, Politzer, Wilczek** receive a Nobel Prize for the discovery of asymptotic freedom in the theory of the strong interactions



¹ There are nine possible color-anticolor combinations, one of which is actually a color singlet: $(\bar{r}r + \bar{g}g + \bar{b}b) / \sqrt{3}$.

Important side note:

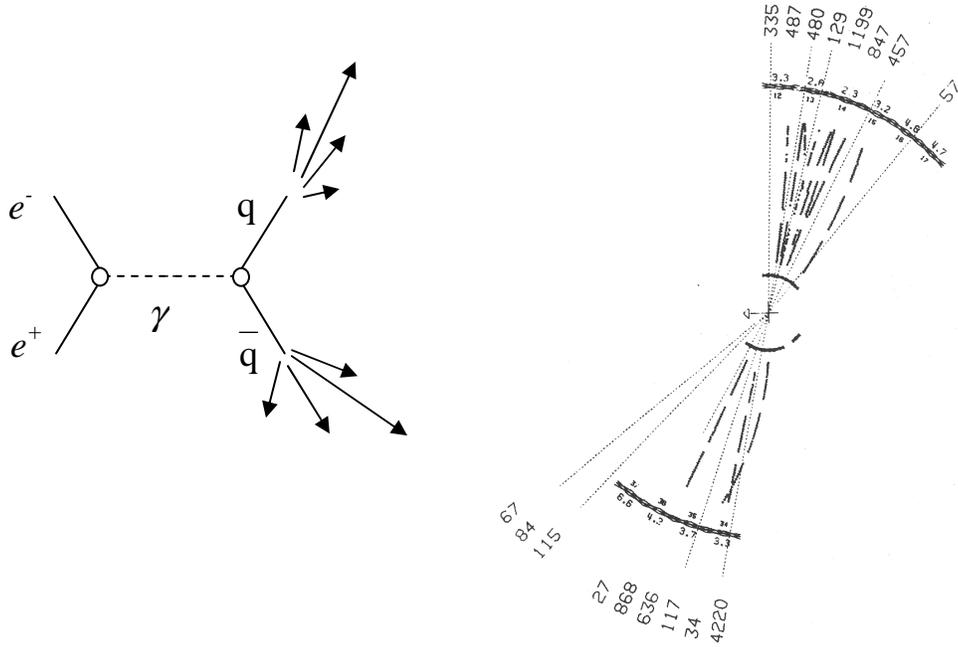
One can conveniently visualize a quark-antiquark pair, as it starts separating after being hit very hard, stretches a string of field between them (a la rubber band). Eventually the rubber band breaks in a few places, which will go into the kinetic energy of smaller pieces. New quark-antiquark pairs are born in the break points and the pieces get isolated from each other. The notion of jets, i.e. sprays of particles flowing in the direction of the original quark/gluon, was born. The higher the energy, the more collimated are jets, which allows their better identification.



The nuclear force between hadrons now can be treated as a remnant of the truly strong force between quarks inside hadrons (a la Van der Waals force between electrically neutral atoms and molecules).

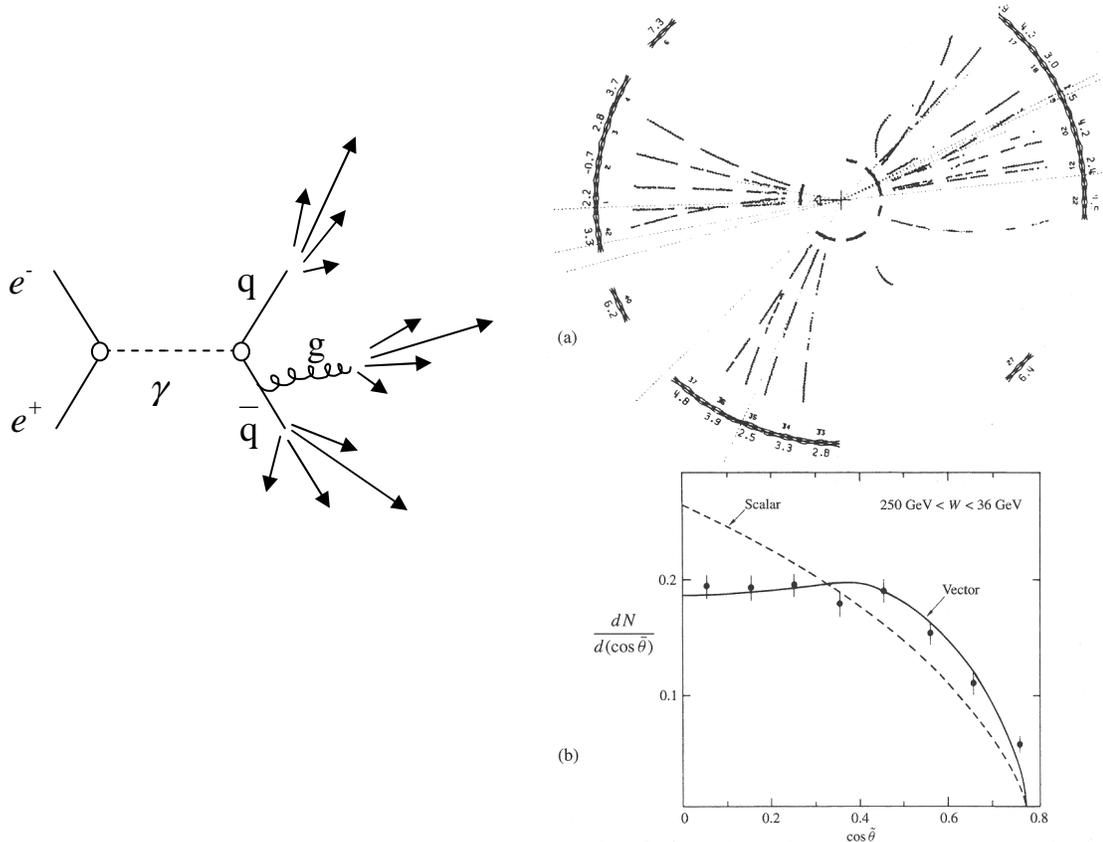
Discovery of gluons

1975 **Hanson** and co-workers at SPEAR, Stanford, report seeing dijets in $e^+e^- \rightarrow$ hadrons. Angular distribution of dijet axis with respect to beam line agrees with quarks being spin-1/2 particles.



The key was to prove that jet-like structure is not a simple statistically-driven combination...

1979 Experiments at PETRA at DESY report 3-jet events. The angular distributions of jets and the rates of such 3-jet events were consistent with the hypothesis of the third jet originating from spin-1 gluons.



ElectroWeak Theory

To avoid nasty quadratic divergences in second-order loop corrections in the four-fermion model of weak interactions, one can suggest that the carrier of the force is massive charged particle. Let's call it W^\pm . Then, the matrix element M will be

$$M = \frac{g^2}{q^2 + M_W^2},$$

which at small energies would appear to be a constant

$$M = \frac{g^2}{M_W^2} = G_F,$$

but at large energies would be similar to the one of QED:

$$M = \frac{g^2}{q^2}.$$

This should hopefully return us back in the realm of logarithmic divergences that we can hide away via the ingenious procedure of renormalization.

BUT #1: This does not quite work. Remember that the gauge invariance, the key for renormalization, requires that the carriers of the force must be massless...

BUT #2: In 1956, it became clear that the weak interactions did not conserve Parity (to be discussed later), which unfortunately implied that not only carriers of the force, but also fermions were not allowed to have masses.

1961 **Glashow** argues that if there were massive W^\pm , there also must be a neutral heavy boson. Let's call it Z^0 .

1964 **Peter Higgs** showed that a combination of 1 field of massless vector (spin-1) bosons and 2 scalar fields (or one complex scalar field ϕ with somewhat unusual potential $V = \lambda(|\phi|^4 - \mu^2|\phi|^2)^2$) can be reinterpreted as 1 massive vector boson and 1 massive scalar. This is a way to sneak in masses in the theory that is built on the gauge invariance principle.

1967 **Weinberg, Salam** independently come up with a self-consistent description of weak interactions mediated by self-interacting vector bosons W^\pm , W , Z^0 , whose masses are acquired via the Higgs mechanism—this means one (or more) additional scalar particle H . In addition, the same Higgs field conveniently allowed one to sneak in masses of fermions! This is done in ad hoc manner and somewhat differently from the case of massive vector bosons, but nevertheless it does do the job. The photon γ was an integral part of the overall picture.

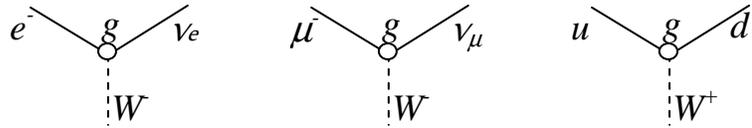
The coupling strength g (up to some mixing angle factors) was the same as in the electromagnetic interactions. None of these assumptions could be departed from without destroying the overall self-consistency.

From $g^2/m^2 = G_F$, one could estimate that mass of charged bosons, W^\pm , had to be of the order of 100 GeV, by far much heavier than what could be produced at the accelerators at that time... They also predicted its lifetime to be very short, $<10^{-24}$ s (plainly due to a huge available phase space).

It is interesting to note that while the electromagnetic part of the EW theory, QED, can exist by itself, while we the EW theory without the photon does not make a self-contained "theory of weak interactions". Nevertheless, for convenience purposes, interactions involving W and Z (either in t- or s-channel) are often referred to as weak interactions.

² A more conventional potential would be $V = \mu^2\phi^2$. The theory is well-behaved only with powers of 2 and 4. Odd powers have no minimum and are not gauge-invariant; while more than 4 powers would make theory not renormalizable.

W^\pm massive bosons that would be responsible for all known weak particle decays and neutrino interactions. Basic vertex diagrams involving charged W-bosons³:



Where $d' = d \cdot \cos\theta + s \cdot \sin\theta$ is a mixture of d and s quarks introduced by Cabbibo to account for all differences in weak decays with and without s-quark involved ($u \leftrightarrow d$ and $u \leftrightarrow s$), which, in addition, could be unified with lepton-neutrino transitions.

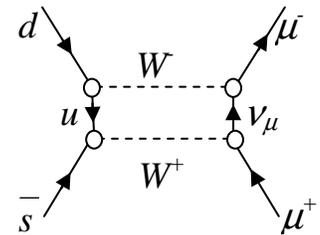
Explain a little better:

- $e\nu W \rightarrow g$
- $\mu\nu W \rightarrow g$
- $udW \rightarrow g'$ (a little smaller than g)
- $usW \rightarrow g''$ (considerably smaller than g)

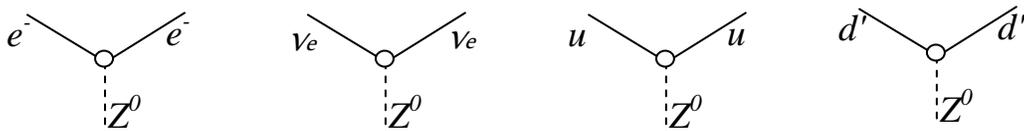
Cabbibo noticed that instead of 3 parameters (g, g', g''), it seemed to be possible to get away with just two: g and θ , so that $g, g'=g\cos\theta, g''=g\sin\theta$. This could be nicely re-interpreted as if W coupled with the full strength g to ud' , where $d' = d \cdot \cos\theta + s \cdot \sin\theta$, and had no sensitivity to us' , where $s' = -d \cdot \sin\theta + s \cdot \cos\theta$ (sate orthogonal to d').

HOWEVER, the theory was clearly missing something.

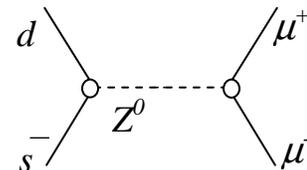
The following second-order diagram would allow transitions between s and d quarks and would result in a number of K-meson decay channels ($K^0 \rightarrow \mu^+ \mu^-$, $K^+ \rightarrow \pi^+ e^+ e^-$) at the rates exceeding the experimental limits. In fact, such decays had never been observed at the time and still have not been observed. Such processes are often referred to as flavor changing neutral currents (FCNC): flavor of quark changes, but the charge does not.



The new electroweak theory required a neutral massive **Z-boson**⁴ with the vertices of the following kind (muon and muon neutrino interaction are copies of the electron-kind ones):



The last diagram appeared to allow transitions between s and d quarks in the first-order and would result in very large branching ratios of FCNC decay modes, e.g., the very same K-meson decay channels.



³ Interactions involving exchange with charged W-bosons are also known as *Charged Current* interactions
⁴ Interactions involving exchange with neutral Z-bosons are also known as *Neutral Current* interactions

1970 **Glashow, Iliopoulos, Maiani** showed⁵ that existence of 4th charm quark would solve all the problems of flavor-changing neutral current decays (both via double-W and Z exchanges!). They also argued on the basis of K_S and K_L ⁶ mass difference that the mass of the 4th quark could not be larger than 3-4 GeV.

The flavor-preserving Neutral Current interactions, however, would remain as processes (obviously, no decays possible). The best channel to look for them would be $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$ since all others will be obscured by much larger electromagnetic or strong interaction cross sections or obscured by charge current weak interactions.

1971-2 **G. 't Hooft and Veltman** proved that the Weinberg-Glashow-Salam theory unifying electromagnetic and weak interaction in one package (with four quarks) was renormalizable.

Weak Interactions:

Carrier of the force: massive spin-1 self-interacting bosons (W^\pm, Z^0)

Particles subject to the force:

- all leptons and quarks
- W and Z bosons themselves
- depending on your taste, you may also include photon as it can split into a W^+W^- pair or can call it an electromagnetic interaction of the charged W-bosons.

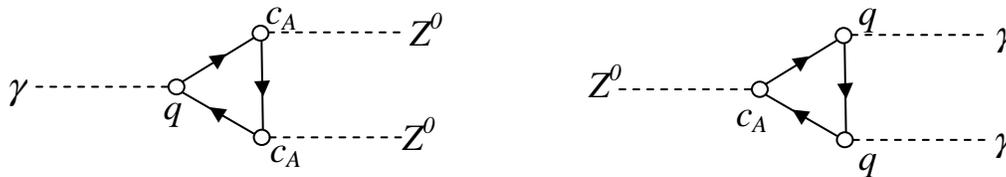
Weak interactions: any interaction involving W and Z bosons

Side Notes:

1964 The idea of the fourth (charm) quark was first put forward by Bjorken and Glashow in 1964, which was based mostly on aesthetic considerations of would-be-good symmetry between 4 leptons and 4 quarks:

leptons	e	ν_e	μ	ν_μ
charge	-1	0	-1	0
quarks	d	u	s	?
charge	-1/3	+2/3	-1/3	+2/3

1972 **Georgi and Glashow** derived a general condition for a gauge theory to be free of anomalies. E.g., the so-called triangle fermion anomalies of the kinds shown below are particularly nasty in SM and must cancel out to keep the electroweak theory renormalizable...



Matrix element of the first one is $M \sim \sum q_i (c_{Ai})^2$, where $c_{A}=\pm 1/2$ for all fermions in the loop (\pm for "up"/"down" fermions in fermion doublets), and, therefore, $M \sim \sum q_i$.

charged fermions	e	μ	uuu	ddd	sss	?
charge	-1	-1	+2	-1	-1	+2

Charm-quark of +2/3 charge, being a partner of the s-quark in the same sense as the u-d pair, and occurring in three colors would do the trick. Note that the same condition dissolves the second diagram as well!

Note that the second diagram shown here has matrix element $M \sim \sum c_{Ai} (q_i)^2$, also, disappears when the fourth quark with charge +2/3 (is included).

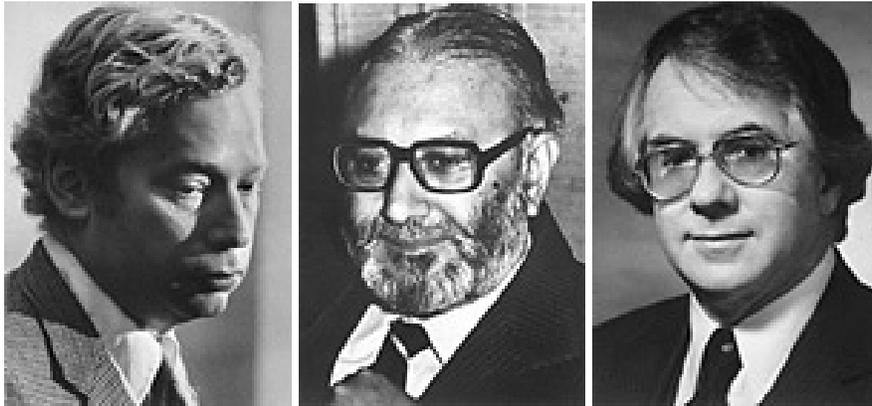
1973 **Kabayashi and Maskawa** showed that by adding 3rd pair of quarks, 5th and 6th, the CP-violation discovered way back in 1964 could be made to be a natural built-in part of the weak interactions⁷.

⁵ So-called GIM mechanism, we will discuss it later

⁶ Mixed states of K^0 and \bar{K}^0 , we will discuss them later.

⁷ To keep the triangle anomaly in bay, this proposition would also imply existence of another pair of leptons!!!

1979 **Weinberg, Salam, Glashow** shared the Nobel Prize "for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current"



1999 **G. 't Hooft, M. Veltman** awarded the Nobel Prize "for elucidating the quantum structure of electroweak interactions in physics"



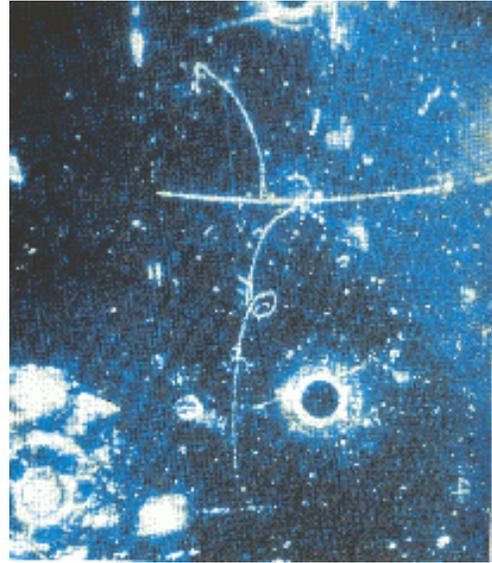
Discovery of Neutral Current (pre-discovery of Z^0)

1973 Old data from Gargamelle, large bubble chamber experiment at CERN, reanalyzed in search for possibly overlooked neutral currents.

Typical processes studied in this experiments earlier were $\nu_\mu + N \rightarrow \mu^- + \text{anything}$

Out of 290,000 emerged 166 events that definitely looked like Neutral Current ones: $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$

Challenge in these studies was to distinguish the NC events from possible background sources, most dangerous/difficult being neutrons kicked out by neutrinos from the material in the chamber metal shell... Nice cross-check is uniformity of the interaction points along the chamber thickness (neutron have relatively large cross-section and would mostly interact at the upstream region of the chamber volume)



After measuring the CC and NC cross sections, the mixing angle θ_w , a parameter in the electroweak theory, could be calculated and masses of W and Z could be very accurately predicted (up to small loop corrections involving top quark and Higgs whose masses were still not known).

Discovery of W^\pm and Z^0

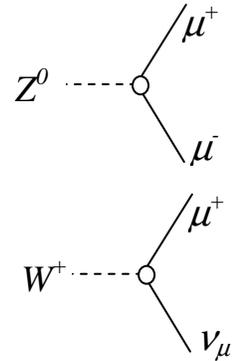
Z- and W-bosons, if directly produced, would decay with well predicted partial widths:

$$Z^0 \rightarrow \bar{u}u, \bar{d}d, e^+e^-, \mu^+\mu^-, \nu_\mu\bar{\nu}_\mu, \dots$$

The cleanest channel to search for is di-lepton pair with the predicted mass $M_Z \sim 90$ GeV

$$W^+ \rightarrow \bar{d}u, \bar{s}u, e^+\nu_e, \mu^+\nu_\mu, \dots$$

The cleanest channel to search for is a high energy lepton ($E > M_W/2$) and missing energy ($E_{\text{miss}} > M_W/2$), where the predicted mass $M_W \sim 80$ GeV



mid-1970

Discussions of what is the best way to reach the desired energies to produce W and Z:

e+ e- collider? The largest available is SPEAR at SLAC with $\sqrt{s}=8$ GeV
 Already under construction PEP at SLAC and PETRA at DESY, both up to 30 GeV
 Would need $\sqrt{s} \sim 90$ GeV to produce Z and $\sqrt{s} \sim 160$ to produce W...
 Synchrotron radiation $\sim (E/m)^4/R^2 \dots$

p \bar{p} collider? None ever built...
 Would be great for W and Z production: $\bar{d} + u \rightarrow W$, $\bar{u} + u \rightarrow Z$
 Naively, would need only ~ 50 GeV per quark, 150 GeV per proton

p p collider? The largest built was ISR, $\sqrt{s}=63$ GeV
 Naively, would need more energy ~ 200 GeV per quark, ~ 600 GeV per proton

p fixed target accelerators? The largest are at CERN (SPS, 400 GeV) and Fermilab (500 GeV)...
 This corresponds to $s^2=2mE$, or $\sqrt{s} \sim 30$ GeV...

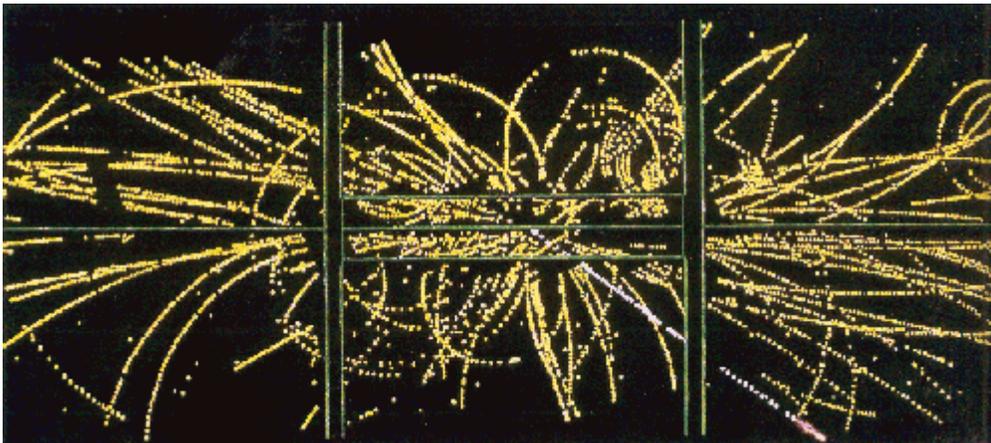
Rubia, Cline, and McIntyre proposed to convert p p machines into p \bar{p} colliders ...

1976 The race begins:

CERN bets on the minimum energy required, SppS with $\sqrt{s}=270+270=540$ GeV, $L \sim 10^{30} \text{ cm}^{-2}\text{s}^{-1}$
 FNAL: first upgrade 500 GeV to 1 TeV; then, convert into p-bar with $\sqrt{s}=1+1=2$ TeV, $L \sim 10^{30} \text{ cm}^{-2}\text{s}^{-1}$

1981 First p-pbar collisions at CERN⁸. Luminosity ($2 \cdot 10^{27}$) was way too low...

1982 CERN reached $2 \cdot 10^{28}$ luminosity, produced 10^9 p-pbar collisions, UA1 and UA2 collaborations collected $\sim 10^6$ events and sifted out 9 events looking like $W \rightarrow e\nu$



⁸ Tevatron turned only in 1987

Important signatures:

- 1) an electron with large transverse momentum
- 2) large imbalance in transverse momentum nicely matching the transverse momentum of the electron
- 3) distribution dN/dp_T has a characteristic shape (so-called Jacobian peak) peaking at larger momentum cut-off: this cutoff corresponds to half-mass of the decaying particle:

COM frame and ignoring spin effects;

A heavy particle of mass m decays into electron and neutrino;

Both electron and neutrino will have $p = m/2$, $p_T = \frac{m}{2} \sin \theta$, $dN / d \cos \theta = const$;

$$\frac{dN}{dp_T} = \frac{dN}{d\left(\frac{m}{2} \sin \theta\right)} = \frac{2}{m} \frac{dN}{d \sin \theta} = \frac{2}{m} \frac{dN}{d \cos \theta} \frac{d \cos \theta}{d \sin \theta} \sim \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{(2p_T / m)}{\sqrt{1 - (2p_T / m)^2}}$$

$$\frac{dN}{dp_T} \sim \frac{(2p_T / m)}{\sqrt{1 - (2p_T / m)^2}}$$

1983 UA1 and UA2 reported the discovery of W, its mass being $\sim 81 \pm 5$ GeV

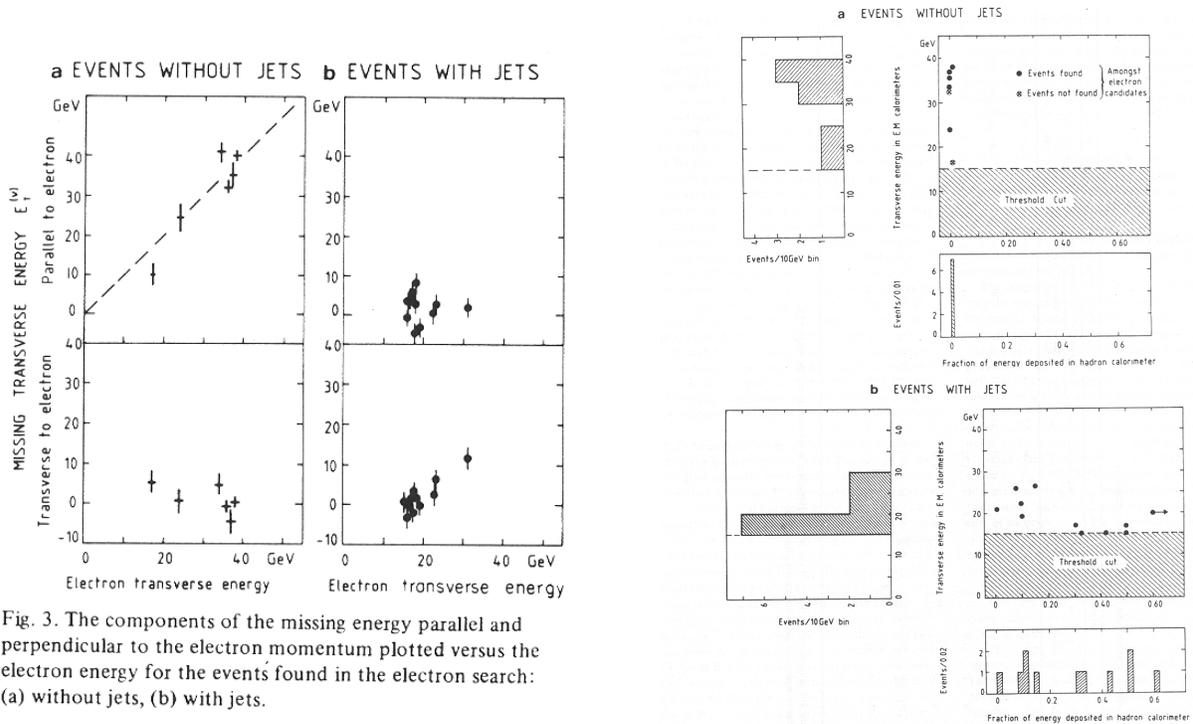
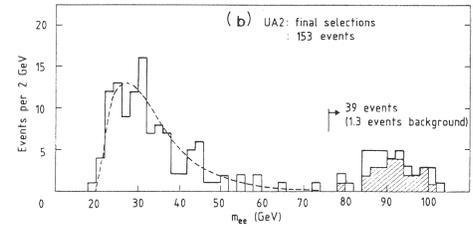
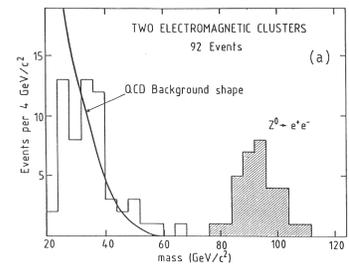
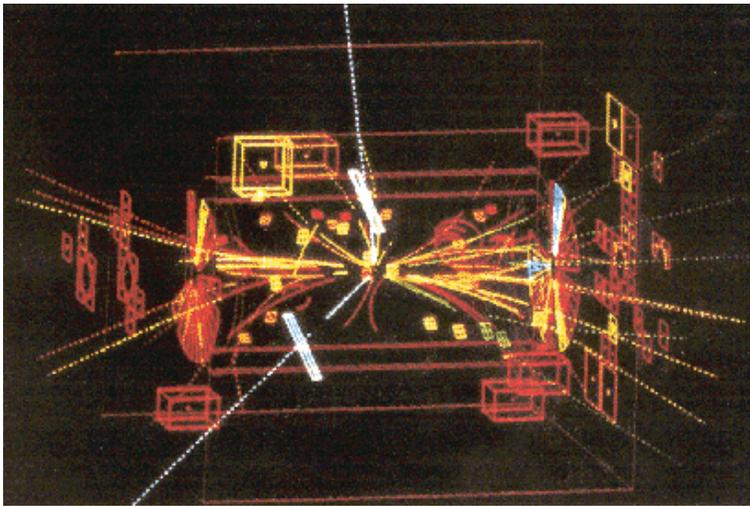


Fig. 3. The components of the missing energy parallel and perpendicular to the electron momentum plotted versus the electron energy for the events found in the electron search: (a) without jets, (b) with jets.

1983 A few months later, UA1 and UA2 reported the discovery of Z, its mass being ~ 91 GeV



1984 **Rubbia** and **Van der Meer** awarded the Noble Prize "for their decisive contributions to the large project, which led to the discovery of the field particles W and Z, communicators of weak interaction"

