Homework 3 *Due on Wednesday Oct 8, 3:00 pm*

18 + 6 (bonus) = 24 points

Problem 1 (1+3+1+1+3=9 points)

Consider a cyclotron of 1930s designed to operate at a fixed frequency. Assume that technology at the time allowed one to provide magnetic field of 1.5 T over the entire cross section of the cyclotron and to apply to the cyclotron dees an alternating e.m.f. in the form of a step wave with ΔV=100 kV.

1) Find the frequency f_0 of the alternating e.m.f to be applied to the cyclotron dees in order to accelerate protons (1 point)
2) Estimate the maximum kinetic energy T_{max} one could accelerate protons to with such a cyclotron (3 points). Hints: At each passing through the gap between dees, proton gains small increment of energy $\varepsilon_0=\Delta V e=0.1$ MeV. Calculate how long it takes the proton to make n half-turns (each turn will be taking different time!) and figure out at what n its arrival to the gap between dees would fall out of the accelerating phase.
3) What must the diameter of such a cyclotron be to reach T_{max}? (1 point)
4) What would happen to protons if the cyclotron diameter was larger? (1 point)
5) To beat the T_{max} limit, what one would need to do? Would it help to increase B, increase ΔV, switch to α-particles. (3 points for answers with correct argumentation)

Problem 2 (1+1=2 points)

What is the most powerful collider in the world at the moment and where is it? (1 point for answering both questions)

What does it accelerate, what is the energy of collisions in the center of mass frame, what is the max luminosity reached so far at this collider? (1 point for answering all three questions)

Problem 3 (1+1+1=3 points)

What will the collision energy be in the Large Hadron Collider? (1 point)

Theorists speculate of a possibility of extra dimensions. In a peculiar scenario when some of the extra dimensions are accessible to gravity only and they are large, creation of tiny black holes might be possible at LHC. Theorists argue that such tiny black holes would evaporate before having any remote chance of growing and swallowing the Earth…

Can you prove from the fact that we still exist, that, if such black holes, indeed, can be made at the LHC, they must impose no danger whatsoever? Hint: estimate the energy needed for a proton colliding with a proton at rest to achieve the same center of mass energy as will be accessible at the LHC (1 point). Use the PDG booklet to find how many cosmic rays of this energy collide with the earth every year (1 point).

Problem 4 (1+1+1+1=4 points)

What were the main ideas behind the large incremental steps in accelerating particles

1) from a few tens MeV earlier cyclotrons to a few hundreds MeV synchrocyclotrons
2) from synchrocyclotrons to earlier a few GeV synchrotrons (Cosmotron, Bevatron, etc.)
3) from the earlier synchrotrons to the ones of a few tens GeV (PS, AGS, etc.)
4) and all the way to 1-TeV beams of the Tevatron

Problem 5 (3+3=6 bonus points)

Consider a high energy primary cosmic ray proton with energy E (e.g., 10^{15} eV) that enters the Earth atmosphere and develops an atmospheric shower. For purposes of these estimates assume that the Earth atmosphere is a layer of N_2 gas of constant density (at NTP) and of a height that would give the normal pressure of 1 bar at the sea level.

Estimate the typical energy of muons reaching the sea level (3 points).
Hints: Consider hadron shower; estimate energy at which pions would start decaying before having a chance to make a nuclear collision; use the fact that $Br(\pi \to \mu\ \nu)$ is almost 100%; check that muons would live long enough to reach the ground level; correct muon energy for dE/dx losses on their way to the ground level.

Estimate the number of muons reaching the sea level (3 points)
Hints: from the full-size PDG book, the average number of charged particles produced in hadron collisions can be approximately parameterized as $\langle n \rangle \sim a \cdot \ln(b \cdot \sqrt{s}/c)$, with $\sqrt{s}$ being the center-of-mass collision energy, a=0.25, b=6, c=200 MeV; follow through the shower development stages all the way to the last stage at which pions would start decaying to muons; remember that about 1/3 of all pions in each stage of multiplication will be π^0's—they must be excluded from further considerations as they would immediately decay to photons.