Probing Anomalous Longitudinal Fluctuations of the Interacting Bose Gas via Bose-Einstein Condensation of Magnons

A. Kreisel, N. Hasselmann, and P. Kopietz, Phys. Rev. Lett. **98**, 067230 (2007)

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Motivation & model

quantum antiferromagnet (QAF)
in magnetic field h

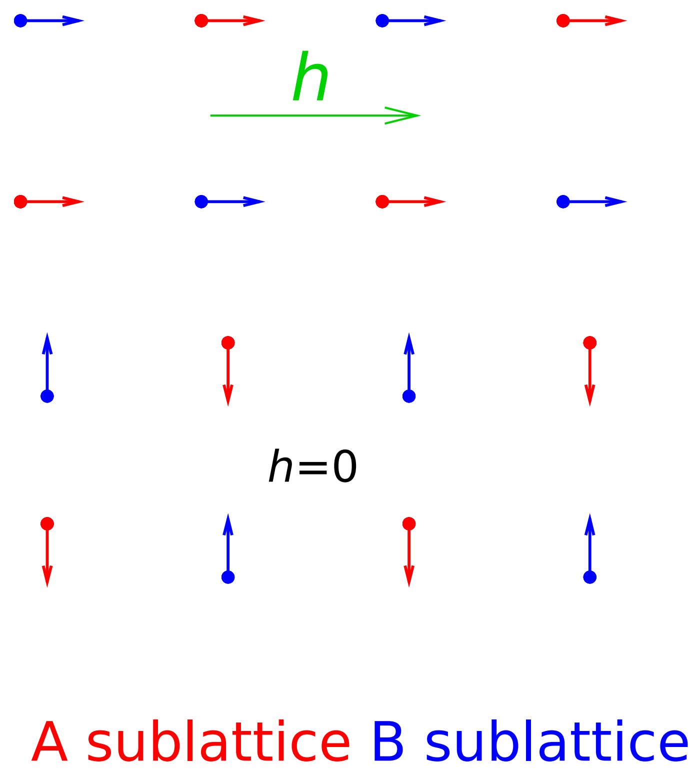
$$H = \frac{1}{2}\sum_{ij} J_{ij}\vec{S}_i \cdot \vec{S}_j - h\sum_i S_i^z$$

- magnons: bosonic quasiparticles
- observation of magnon BEC possible
  experiment using Cs_2CuCl_4 [1]

1. $h > h_c$ ferromagnetic order
   $U(1)$-symmetric ground state
2. $h < h_c$ antiferromagnetic order
   $U(1)$-symmetry spontaneously broken $\hat{=}$ MBEC

Classical ground states of QAF

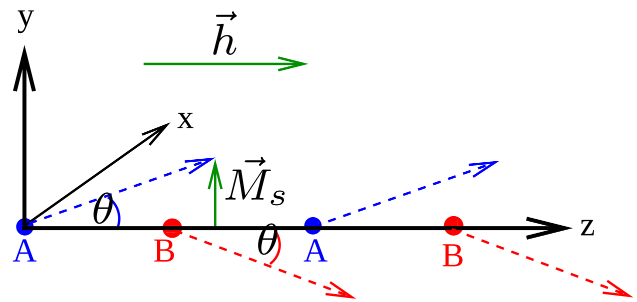
in a magnetic Field



[1] T. Radu, H. Wilhelm, V. Yushankhai, D. Kovrizhin, R. Coldea, Z. Tylczynski, T. Lühmann, and F. Steglich, Phys. Rev. Lett. **95**, 127202 (2005).

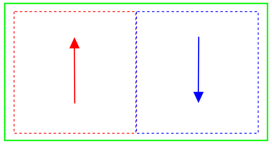
Formulation in terms of magnons

- Holstein-Primakoff-transformation of Heisenberg modell along axes of FM ground state [1]
  $\longrightarrow$ bosons $\left[b_i, b_i^\dagger\right] = 1$



- fourier transformation: reduced Brillouinzone
  $\longrightarrow$ 2 Modes

$$H_2 = \sum_{\vec{k}\sigma=\pm 1} \left(\epsilon_{\vec{k}\sigma} - \mu\right) b^\dagger_{\vec{k}\sigma} b_{\vec{k}\sigma}$$



AF unit cell in real space

- neglect gapped mode, continuum formulation $\longrightarrow$ interacting bosonic fields

$$H = \int \frac{d^D\vec{k}}{(2\pi)^D} \left(\frac{\vec{k}^2}{2m} - \mu \right) \Psi^\dagger_{\vec{k}} \Psi_{\vec{k}} + \frac{1}{2} \int_{\vec{k}} \int_{\vec{k}'} \int_{\vec{q}} U_{\vec{q}} \Psi^\dagger_{\vec{k}+\vec{q}} \Psi^\dagger_{\vec{k}'-\vec{q}} \Psi_{\vec{k}'} \Psi_{\vec{k}}$$

$U_{\vec{q}} = \Theta(\Lambda_0 - |\vec{q}|)\chi_0^{-1}$ effective interaction $\qquad \chi_0 = (2\tilde{J}_0 a^D)^{-1}$

[1] E. G. Batyev and L. S. Braginskii, Zh. Eksp. Teor. Fiz. **87**, 1361 (1984) [Sov. Phys. JETP **60**, 781 (1984)

Hermitian parametrization

- hermitian operators $\Pi_{\vec{k}} = \Pi^{\dagger}_{-\vec{k}}$, $\Phi_{\vec{k}} = \Phi^{\dagger}_{-\vec{k}}$ $\left[\Pi_{\vec{k}}, \Phi_{\vec{k}'}\right] = i(2\pi)^D \delta(\vec{k} - \vec{k}')$

$$\Psi_{\vec{k}} = \sqrt{\frac{s}{2}}\theta \Pi_{\vec{k}} + \frac{i}{\sqrt{2s\theta}}\Phi_{\vec{k}}$$ [1] $s = S/a^D$ spin-density

- advantace: physical interpretation of operators as staggered spin fluctuations in the directions perpendicular to h

$$\Pi_{\vec{k}} \sim \sum_i \zeta_i e^{-i\vec{k}\cdot\vec{r}_i} S_i^x$$

$$\Phi_{\vec{k}} \sim \sum_i \zeta_i e^{-i\vec{k}\cdot\vec{r}_i} S_i^y$$

$$\zeta_i = \begin{cases} 1 & i \in A \\ -1 & i \in B \end{cases}$$

[1] N. Hasselmann and P. Kopietz, Europhys. Lett. **74**, 1067 (2006).

Symmetry breaking

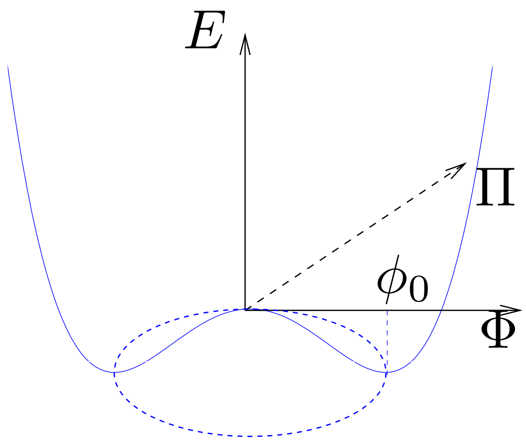
$$\Phi_{\vec{k}} = \Delta\Phi_{\vec{k}} + (2\pi)^D \delta(\vec{k})\phi_0$$

$U(1)$-symmetry broken

$$\Rightarrow H = E_0 + H_1 + H_2 + H_3 + H_4$$

$$\mu = \frac{U_0 s \phi_0}{2}$$

$H_1 = 0$ fixes chemical potential



diagonalization of quadratic part: dispersion of antiferromagnetic magnons

$$\epsilon_{\vec{k}} = |\vec{k}|c_0 + \mathcal{O}(\vec{k}^3) \qquad c_0 = 2\theta J a\sqrt{D}$$

Correlations

using results of [1] yields correlations $K = (\vec{k}, i\omega)$

$$\langle \Pi_K \Pi_{K'} \rangle = \delta_{K,-K'} \frac{\chi_0^{-1}}{\omega^2 + c_0^2 \vec{k}^2} \qquad \langle \Pi_K \Phi_{K'} \rangle = \delta_{K,-K'} \frac{\omega}{\omega^2 + c_0^2 \vec{k}^2}$$

$$\langle \Phi_K \Phi_{K'} \rangle = \delta_{K,-K'} \frac{\chi_0 c_0^2 \vec{k}^2}{\omega^2 + c_0 \vec{k}^2}$$

quantitatively wrong
in gaussian approximation

$$\langle \Phi_K \Phi_{K'} \rangle = \delta_{K,-K'} \chi \left(\frac{-Z_\parallel^2 \omega^2}{\omega^2 + c^2 \vec{k}^2} + K_{D+1} \frac{(mc)^3}{Z_\rho^2 \rho_0} \begin{cases} \ln \left[\frac{(mc)^2}{\omega^2/c^2 + \vec{k}^2} \right] & D = 3 \\ \frac{2}{3-D} \left(\frac{\omega^2}{c^2} + \vec{k}^2 \right)^{\frac{D-3}{2}} & 1 < D < 3 \end{cases} \right)$$

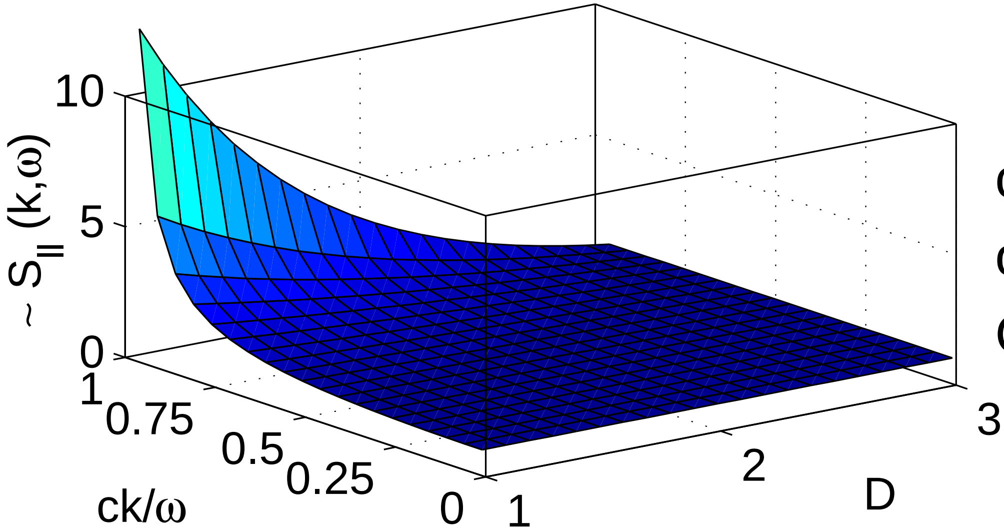
exact result with nonanalytic term: resummation of infrared divergences
= anomalous dimension in $\langle \Phi_K \Phi_{K'} \rangle$

[1] C. Castellani, C. Di Castro, F. Pistolesi, and G. C. Strinati, Phys. Rev. Lett. **78**, 1612 (1997); F. Pistolesi, C. Castellani, C. Di Castro, and G. C. Strinati, Phys. Rev. B **69**, 024513 (2004).

possible measurement

- neutron scattering might detect anomalous dimension
- longitudinal staggered structure factor

$$S_{\|}(\vec{k},\omega) = \frac{\chi s^2}{M_s^2}\left[\frac{Z_{\|}^2}{2} c|\vec{k}|\delta(\omega - c|\vec{k}|) + \frac{C_D (mc)^3 \Theta(\omega - c|\vec{k}|)}{Z_\rho^3 \rho_0 \left(\frac{\omega^2}{c^2} - \vec{k}^2\right)^{\frac{3-D}{2}}}\right]$$



critical continuum

only valid at $\frac{\omega}{c} \leq k_G$

Ginzburg-scale

Restrictions

- Ginzburg-scale: $\frac{\omega}{c} \leq k_G$

$$k_G \approx \frac{(mc)^3}{\rho_0} \approx \frac{4\sqrt{2}}{Sa}\theta \sim \theta \qquad D = 2$$

$$k_G \approx mc\, e^{-\frac{\rho_0}{(mc)^3}} \approx \frac{\sqrt{3}}{a}\theta e^{-\frac{S}{6\sqrt{3}\theta}} \qquad D = 3$$

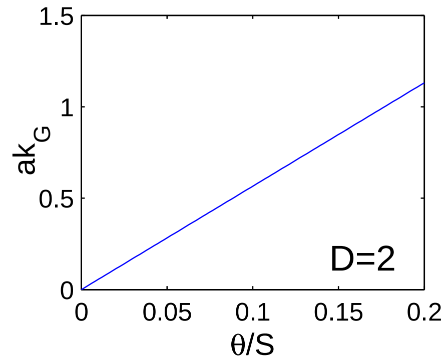
$$\theta \sim \sqrt{h_c - h} \quad \text{tilt angle}$$

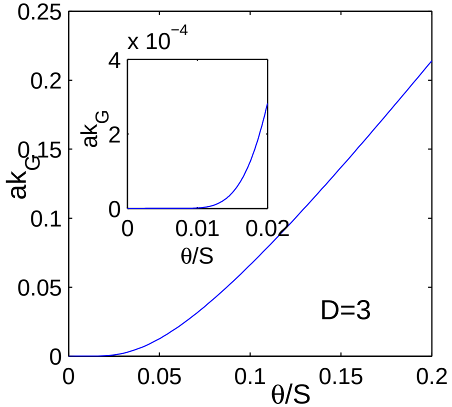
- energy integrated structure factor (spectral weight)

I_δ: δ-function $\quad$ I_c: critical continuum part

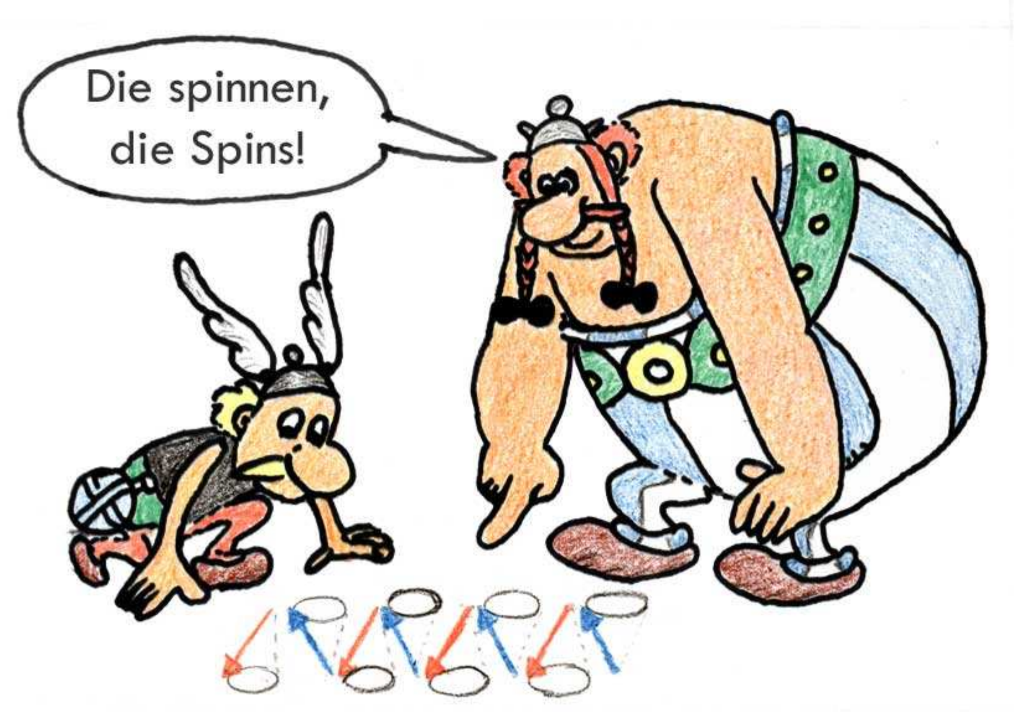
$$\frac{I_c}{I_\delta} \approx \frac{k_G}{|\vec{k}|} \ln\left(\frac{k_G}{|\vec{k}|}\right) \qquad D = 2$$

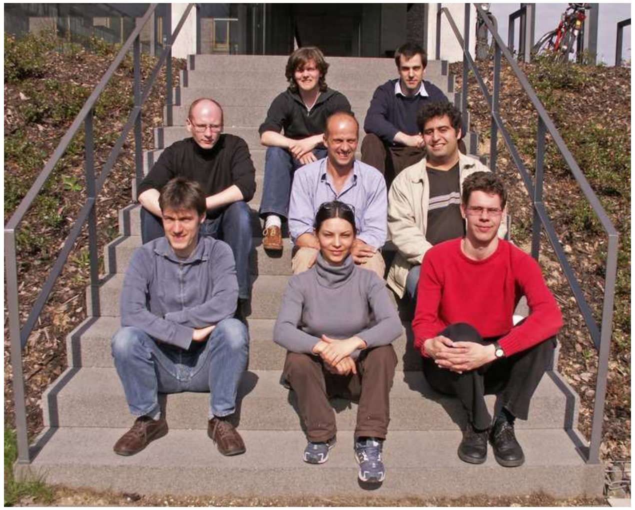
$$\frac{I_c}{I_\delta} \approx \frac{(mc)^3}{\rho_0} \frac{k_G}{|\vec{k}|} \qquad D = 3$$





Summary & acknowledgement





group of Peter Kopietz

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