VECTOR DEFINITIONS, IDENTITIES, AND THEOREMS

DEFINITIONS

Rectangular Coordinates

1. $$\nabla f = \frac{\partial f}{\partial x}\boldsymbol{i} + \frac{\partial f}{\partial y}\boldsymbol{j} + \frac{\partial f}{\partial z}\boldsymbol{k}$$

2. $$\nabla \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

3. $$\nabla \times \boldsymbol{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\boldsymbol{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\boldsymbol{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\boldsymbol{k}$$

4. $$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

5. $$\nabla^2 \boldsymbol{A} = \nabla^2 A_x \boldsymbol{i} + \nabla^2 A_y \boldsymbol{j} + \nabla^2 A_z \boldsymbol{k}$$

Cylindrical Coordinates

1. $$\nabla f = \frac{\partial f}{\partial \rho}\boldsymbol{\rho}_1 + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\boldsymbol{\varphi}_1 + \frac{\partial f}{\partial z}\boldsymbol{z}_1$$

2. $$\nabla \cdot \boldsymbol{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

3. $$\nabla \times \boldsymbol{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\boldsymbol{\rho}_1 + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\boldsymbol{\varphi}_1 + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}(\rho A_\varphi) - \frac{\partial A_\rho}{\partial \varphi}\right)\boldsymbol{z}_1$$

4. $$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

5. $\nabla^2 \boldsymbol{A} = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla \times \nabla \times \boldsymbol{A}$ (See Section 1.9.6.)

Spherical Coordinates

1. $$\nabla f = \frac{\partial f}{\partial r}\boldsymbol{r}_1 + \frac{1}{r}\frac{\partial f}{\partial \theta}\boldsymbol{\theta}_1 + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\boldsymbol{\varphi}_1$$

2. $$\nabla \cdot \boldsymbol{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

3. $$\nabla \times \boldsymbol{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\varphi \sin\theta) - \frac{\partial A_\theta}{\partial \varphi}\right]\boldsymbol{r}_1 + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial (rA_\varphi)}{\partial r}\right]\boldsymbol{\theta}_1 + \frac{1}{r}\left[\frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\boldsymbol{\varphi}_1$$

4. $$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$$

5. $\nabla^2 \boldsymbol{A} = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla \times \nabla \times \boldsymbol{A}$ (See Section 1.9.6.)

IDENTITIES

1. $\nabla(fg) = f\nabla g + g\nabla f$

2. $\nabla(\boldsymbol{A}\cdot\boldsymbol{B}) = (\boldsymbol{B}\cdot\nabla)\boldsymbol{A} + (\boldsymbol{A}\cdot\nabla)\boldsymbol{B} + \boldsymbol{B}\times(\nabla\times\boldsymbol{A}) + \boldsymbol{A}\times(\nabla\times\boldsymbol{B}),$

3. $\nabla\cdot(f\boldsymbol{A}) = (\nabla f)\cdot\boldsymbol{A} + f(\nabla\cdot\boldsymbol{A}),$

4. $\nabla\cdot(\boldsymbol{A}\times\boldsymbol{B}) = \boldsymbol{B}\cdot(\nabla\times\boldsymbol{A}) - \boldsymbol{A}\cdot(\nabla\times\boldsymbol{B}),$

4. $\nabla\times(f\boldsymbol{A}) = (\nabla f)\times\boldsymbol{A} + f(\nabla\times\boldsymbol{A}),$

5. $\nabla\times(\boldsymbol{A}\times\boldsymbol{B}) = (\boldsymbol{B}\cdot\nabla)\boldsymbol{A} - (\boldsymbol{A}\cdot\nabla)\boldsymbol{B} + (\nabla\cdot\boldsymbol{B})\boldsymbol{A} - (\nabla\cdot\boldsymbol{A})\boldsymbol{B}.$

6. $\nabla\times\nabla\times\boldsymbol{A} = \nabla(\nabla\cdot\boldsymbol{A}) - \nabla^2\boldsymbol{A}$ (See Section 1.9.6.)

7. $$(\boldsymbol{A}\cdot\nabla)\boldsymbol{B} = \left[A_x\frac{\partial B_x}{\partial x} + A_y\frac{\partial B_x}{\partial y} + A_z\frac{\partial B_x}{\partial z}\right]\boldsymbol{i} + \left[A_x\frac{\partial B_y}{\partial x} + A_y\frac{\partial B_y}{\partial y} + A_z\frac{\partial B_y}{\partial z}\right]\boldsymbol{j} + \left[A_x\frac{\partial B_z}{\partial x} + A_y\frac{\partial B_z}{\partial y} + A_z\frac{\partial B_z}{\partial z}\right]\boldsymbol{k}$$

8. $\nabla'\left(\frac{1}{r}\right) = \frac{\boldsymbol{r}_1}{r^2}$, where the gradient is calculated at the source point (x', y', z') and $\boldsymbol{r}_1$ is the unit vector from the source point (x', y', z') to the field point (x, y, z).

9. $\nabla\left(\frac{1}{r}\right) = -\frac{\boldsymbol{r}_1}{r^2}$, where the gradient is calculated at the field point, with the same unit vector.

10. $\int_\tau \nabla f\, d\tau = \int_S f\, d\boldsymbol{a}$

11. $\int_\tau (\nabla\times\boldsymbol{A})\, d\tau = \int_S \boldsymbol{A}\times d\boldsymbol{a},$

    where S is the surface which bounds the volume τ.

THEOREMS

1. Divergence theorem: $\int_S \boldsymbol{A}\cdot d\boldsymbol{a} = \int_\tau \nabla\cdot\boldsymbol{A}\, d\tau,$

   where S is the surface which bounds the volume τ.

2. Stokes's theorem: $\oint_C \boldsymbol{A}\cdot d\boldsymbol{l} = \int_S (\nabla\times\boldsymbol{A})\cdot d\boldsymbol{a},$

   where C is the closed curve which bounds the surface S.

CONVERSION TABLE

Examples: One meter equals 100 centimeters. One volt equals 10^8 electromagnetic units of potential.

	SI	CGS SYSTEMS	
		esu	emu
Length	meter	10^2 centimeters	10^2 centimeters
Mass	kilogram	10^3 grams	10^3 grams
Time	second	1 second	1 second
Force	newton	10^5 dynes	10^5 dynes
Energy	joule	10^7 ergs	10^7 ergs
Power	watt	10^7 ergs/second	10^7 ergs/second
Charge	coulomb	3×10^9	10^{-1}
Electric potential	volt	1/300	10^8
Electric field intensity	volt/meter	$1/(3 \times 10^4)$	10^6
Electric displacement	coulomb/meter^2	$12\pi \times 10^5$	$4\pi \times 10^{-5}$
Displacement flux	coulomb	$12\pi \times 10^9$	$4\pi \times 10^{-1}$
Electric polarization	coulomb/meter^2	3×10^5	10^{-5}
Electric current	ampere	3×10^9	10^{-1}
Conductivity	mho/meter	9×10^9	10^{-11}
Resistance	ohm	$1/(9 \times 10^{11})$	10^9
Capacitance	farad	9×10^{11}	10^{-9}
Magnetic flux	weber	1/300	10^8 maxwells
Magnetic induction	tesla	$1/(3 \times 10^6)$	10^4 gausses
Magnetic field intensity	ampere/meter	$12\pi \times 10^7$	$4\pi \times 10^{-3}$ oersted
Magnetomotance	ampere	$12\pi \times 10^9$	$(4\pi/10)$ gilberts
Magnetic polarization	ampere/meter	$1/(3 \times 10^{13})$	10^{-3}
Inductance	henry	$1/(9 \times 10^{11})$	10^9
Reluctance	ampere/weber	$36\pi \times 10^{11}$	$4\pi \times 10^{-9}$

NOTE: We have set $c = 3 \times 10^8$ meters/second.