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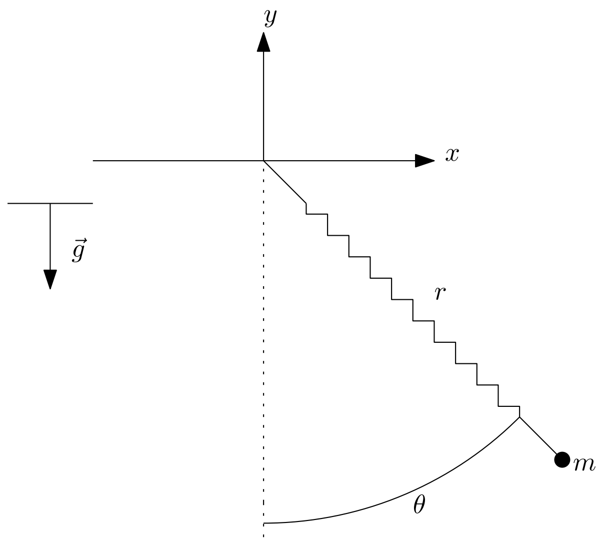
# TP 3: Classical Mechanics 2 and Electrodynamics 2 — Sheet 2

*Winter Term 2019/2020*

**Due date:** Friday, 01.11.2019, 1pm. Solve the exercises marked with a star* and hand in your solution into the mailbox for TP3. Prepare the other exercises *at home* such that you will be able to discussed these in the exercise classes of week 45: Tuesday 05.11.2019 or Thursday 07.11.2019.

## 1. Spring Pendulum*

In a spring pendulum, a mass $m$ is hanging on a spring vertically in the gravitational field. Assume that the mass $m$ can only move in the drawing plane, see figure below.




The rest length of the spring (when no forces act on it) is $l$.

a) How many degrees of freedom are in this system? Explain how you came to this conclusion. Give two possible choices of generalized coordinates $\vec{q}(t)$ for this system and justify which one you are choosing.

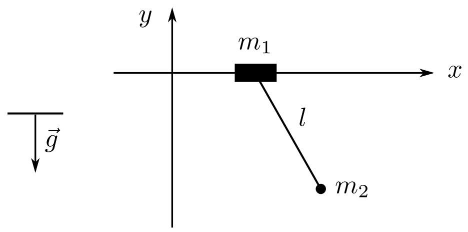
b) Write down the position and the velocity vector ($\vec{r}$ and $\dot{\vec{r}}$) using the generalized coordinates. Make a suitable choice of your coordinate system.

c) Write down the kinetic energy $T$ and potential energy $V$ of the system in terms of the generalized coordinates and use the Euler-Lagrange equations to obtain the equations of motion.

d) Solve the equations of motion for small deviations from the equilibrium postion.

## 2. Crane pendulum

A pendulum with length $l$ and mass $m_2$ is suspended from the trolley (of mass $m_1$) of an overhead travelling crane which can move freely in horizontal direction. Derive the Lagrange function of this system and the equations of motion.




## 3. Mechanical gauge invariance of the Lagrange equations

Consider a change of the Lagrangian,

$$L'(q, \dot{q}, t) = cL(q, \dot{q}, t) + \frac{d}{dt}F(q, t),$$

where $c$ is a constant but nonzero factor and the second term is a total time derivative of a function $F = F(q, t)$ which only depends on $q$ and $t$. Prove the statement that both Lagrange functions yield the same equations of motion by inserting $L'(q, \dot{q}, t)$

a) directly into the Euler-Lagrange equations.

b) into the Hamilton's principle.