Institut für Theoretische Physik
Universität Leipzig

Dr. A. Kreisel
Rohan Kulkarni

TP 3: Classical Mechanics 2 and Electrodynamics 2 — Sheet 3

Winter Term 2019/2020

Due date: Friday, 08.11.2019, 1pm. Solve the exercises marked with a star* and hand in your solution into the mailbox for TP3. Prepare the other exercises *at home* such that you will be able to discussed these in the exercise classes of week 45: Tuesday 12.11.2019 or Thursday 14.11.2019.

1. Noether Theorem*

Consider a particle with mass m that undergoes a free fall in the (homogeneous) gravitational field.

a) Assuming that the particle only moves in the vertical direction, find the Lagrange function $L(q, \dot{q}, t)$ for this system. Explain how you have chosen your coordinate system.

b) Write down the equations of motion as obtained from $L(q, \dot{q}, t)$ and solve the resulting differential equation for $q(t)$ and $\dot{q}(t)$.

c) Consider the transformation $z \to z' = z + vt$ and show that it is a symmetry transformation of the system. How is such a transformation of the coordinate system called?
Hint: Consider the result of Exercise 3, sheet 2 and find the corresponding function $F(q, t)$.

d) Calculate the conserved quantity according to Noether's theorem

$$I(q, \dot{q}, t) = \frac{\partial L}{\partial \dot{q}} \frac{\partial q}{\partial \alpha}\bigg|_{\alpha=0} - \frac{\partial F}{\partial \alpha}\bigg|_{\alpha=0} .$$

Hint: Use the result of the previous task and identify the parameter α in the symmetry transformation.
Explain how you can verify that $I(q, \dot{q}, t)$ is indeed constant.

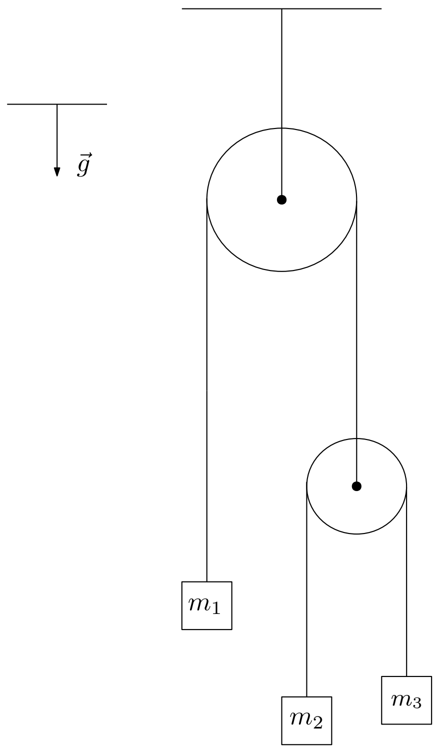
2. Lagrange equations of first kind

A massless, inextensible string passes over a massless smooth pulley and carries a mass $m_1 = 4m$ on one end. The other end supports a second massless pulley with a string over it carrying masses $m_2 = 3m$ and $m_3 = m$ on the two ends.

a) Use appropriate generalized coordinates and write down the Lagrangian of the system.

b) Using the Euler Lagrange equation, find the downward accelerations of the three masses m_1, m_2 and m_3.

c) Using the method of Lagrange multipliers, find the tensions in the strings for those particular masses. Explain why the first pulley turns even though the total mass on each side is the same.



3. Legendre transformation

a) Calculate the Legendre transform h of the functions $f(x) = x^3$ and $f(x) = \exp(x)$ with respect to x. Check your results by calculating the product of the second derivatives $f''h'' = 1$ and give a region of validity.

b) The Legendre transformation is also applied in thermodynamics where the internal energy of a gas $E(S, V)$ depends on the entropy S and the volume V and has the total differential $dE = TdS - PdV$. Construct the following Legendre transforms of the internal energy $E(S, V)$

   (a) enthalpy $H(S, P)$ as the Legendre transform of $E(S, V)$ with respect of V

   (b) free energy $F(T, V)$ as Legendre transform of $E(S, V)$ with respect of S

   (c) Gibbs free energy $G(T, P)$

   Hint use the definition of the Legendre transform and read off the partial derivatives from the total differentials.