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TP 3: Classical Mechanics 2 and Electrodynamics 2 — Sheet 4

Winter Term 2019/2020

Due date: Friday, 15.11.2019, 1pm. Solve the exercises marked with a star* and hand in your solution into the mailbox for TP3. Prepare the other exercises *at home* such that you will be able to present these at the blackboard in the exercise classes of week 46: Tuesday 19.11.2019 or Thursday 21.11.2019.

1. Saddle point of the action

Show that the action

$$S = \int_0^{t_2} \left[\frac{m}{2}\dot{x}_0^2 - \frac{k}{2}x_0^2 \right] dt$$

of the harmonic oscillator with spring constant k for the trajectory

$$x_0(t) = a\sin(\omega_0 t)$$

with $\omega_0 = \sqrt{\frac{k}{m}}$ is is neither minimal nor maximal if t_2 is larger than the half period $T/2$.
Instructions: Use the ansatz

$$x_\alpha(t) = x_0(t) + \alpha\eta(t)$$

with $n(0) = n(t_2) = 0$ and insert it into the action S, collect the terms according to the powers of α. Show that the terms linear in α vanish by use of integration by parts and rewrite also the terms proportional to α^2 using integration by parts. Next, make the ansatz for the variational function

$$\eta(t) = \sum_{k=1}^{\infty} b_k \sin\left(\frac{k\pi}{t_2}t\right) ,$$

where b_k are arbitrary constants and analyse the result.

2. Phase space trajectories*

a) Find the equation of motion of the mathematical pendulum (mass on massless stick, moving in a plane) using the Newton, the Lagrange and the Hamilton formulation of classical mechanics. (Do not make the approximation for small angles.) Describe the quantities that are introduced the mathematical operations that are done.

b) Draw and discuss the trajectories in phase space of the pendulum for the following cases:

  (a) The maximal value of the angle ϕ_0 is smaller than π.
  (b) The maximal value of the angle ϕ_0 is equal π.
  (c) The pendulum does rollovers.

3. Poisson brackets*

Write down the Poisson brackets between the angular momentum $\vec{L} = \vec{r} \times \vec{p}$ and an arbitrary vector field that depends on the position and momentum $\vec{A}(\vec{r}, \vec{p})$. Explain which calculation rules for the Poisson brackets can be used to calculate Poisson brackets between vector quantities.

Using this relation calculate the following Poisson brackets

a) $\{L_i, r_j\}$

b) $\{L_i, p_j\}$

c) $\{L_i, L_j\}$

Hint: Use the expression $L_i = (\vec{r} \times \vec{p})_i = \epsilon_{ijk} r_j p_k$ for the components of the angular momentum. (Note that the Einstein's sum convention was used, i.e. indices appearing twice in a product are summed over. The explicit summation sign $\sum_{ij}$ is omitted. You can use the the fundamental Poisson brackets from the lecture, or just work out the partial derivatives in a specific set of canonical variables. State the steps you are performing and which calculation rules you are using.