Institut für Theoretische Physik
Universität Leipzig

Dr. A. Kreisel
Rohan Kulkarni

TP 3: Classical Mechanics 2 and Electrodynamics 2 — Sheet 6

Winter Term 2019/2020

Due date: Friday, 29.11.2019, 1pm. Solve the exercises marked with a star* and hand in your solution into the mailbox for TP3. Prepare the other exercises *at home* such that you will be able to present these at the blackboard in the exercise classes of week 49: Tuesday 03.12.2019 or Thursday 05.12.2019.

1. Coupled harmonic oscillators*

The quantum mechanical Hamiltonian

$$\hat{H} = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_2^2}{2m} + \frac{m\omega_0^2}{2}(\hat{x}_1^2 + \hat{x}_2^2) + mK\hat{x}_1\hat{x}_2$$

can be used to derive the attactive force between neutral atoms of mass m ("Van de Waals force"). K is a parameter for the coupling. Consider the classical version of the Hamiltonian and calculate the eigenfequencies of the corresponding classical system.

Guidance: First, obtain the Lagrange function $L(\vec{q}, \dot{\vec{q}}, t)$ of the system and bring it into the form $L = \dot{\vec{q}}^T M \dot{\vec{q}} - \vec{q}^T K \vec{q}$ with two matrices M and K and the vector $\vec{q}$ of the generalized coordinates. Then perform a suitable orthogonal transformation to make the corresponding matrices diagonal and read off the eigenfrequencies from the equations of motion.

2. Hamilton function for driven system*

A particle of mass m is subject to the homogeneous gravitational field and bound on the surface of a sphere with radius $R(t)$ which changes as a given function of time.

a) Obtain the Hamilton function of the system and derive the corresponding canonical equations of motion.

b) Are there any integrals of motion in this system?

c) Argue why or why not the Hamilton function is the total energy $E = T + V$ of the system.

3. Canonical transformation

Given a Hamilton function of a system which is a sum of a time-dependent contribution and a time-independent contribution

$$H(q,p,t) = H_0(q,p) - aq\sin(\omega t)\,,$$

where a and ω are constants.

a) Derive the Hamilton equations of motion.

b) Obtain a canonical transformation, which brings the Hamiltonian into a form without explicit time-dependence.
Hint: Use the transformation formula for the Hamilton function upon use of a generating function F to read off a differential equation for F. The solution then induces the desired transformation.