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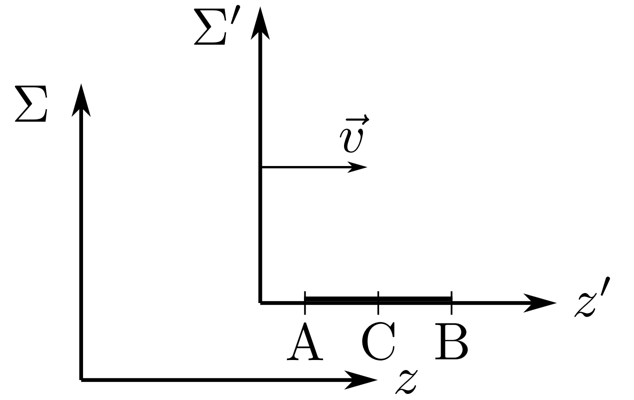
TP 3: Classical Mechanics 2 and Electrodynamics 2 — Sheet 7

Winter Term 2019/2020

Due date: Friday, 06.12.2019, 1pm. Solve the exercises marked with a star* and hand in your solution into the mailbox for TP3. Prepare the other exercises *at home* such that you will be able to present these at the blackboard in the exercise classes of week 50: Tuesday 10.12.2019 or Thursday 12.12.2019.

1. Invariance of the velocity of light and relative simultaneousness

Argue upon the following gedankenexperiment that the postulate of the invariance of the velocity of light is incompatible with the absolute simultaneousness of events.



For this, consider two reference frames Σ and Σ', the latter moving with relative velocity $\vec{v}$ compared to Σ. There is a straight-line segment AB parallel to $\vec{v}$ with the center point C which rests in Σ'. Now, assume the invariance of the velocity of light and the absolute time: A light signal is generated in point C. Explain what conclusions observers in the reference frame Σ and Σ' draw about the arrival times in points A and B?

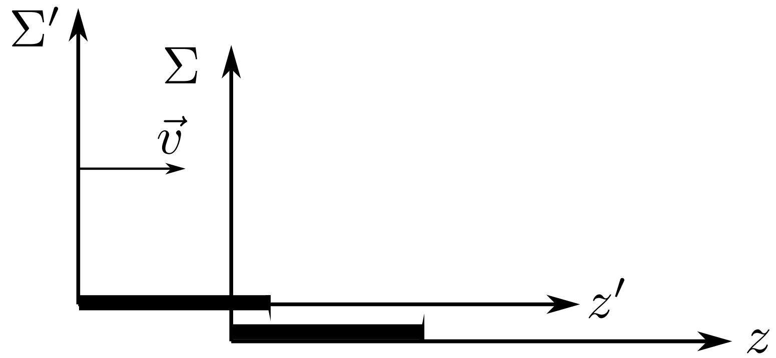
2. Invariant intervals and the metric tensor

a) Write down the invariant interval for the following spaces $\mathbb{R}^2$, $\mathbb{R}^3$, $\mathbb{R}^4$ and name the transformation group that leaves the interval invariant.

b) Given the metric tensor $g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ and the relation of contravariant and covariant 4-vectors. Calculate $g^{\mu\nu}$, $g_\nu{}^\mu$, and $(g^{-1})^{\mu\nu}$. In Minkowski space, the interval $ds^2 = c^2dt^2 - \sum_i (dx_i)^2$ is invariant. Use this to show that also $g_{\mu\nu}$ is invariant under Lorentz transformations.

3. Measurement of the length of a rod*

Consider two reference systems Σ and Σ', the latter is moving relatively with constant velocity $v = 0.6c$. In each system, there is a rod of the same (resting) length fixed with one end at the origin of the respective coordinate system. Each rod has a needle at one end that can be used to mark the other rod. Once the starting position of the two rods agree in the system Σ, the marking mechanism is released such that it can mark the other rod if it is under the needle. (In the drawing, the needle on the rod in Σ' could mark the rod in Σ, but the needle on the rod in Σ would not mark anything.)



Assume that the coordinate origins are identical at $t = t' = 0$ and the marking mechanisms are released at the time $t = 0$ as viewed from the resting coordinate system Σ.

a) Which rod is shorter (as seen from the resting observer in Σ) and which rod is marked? Where?

b) Which rod is shorter as seen from an observer in the other reference frame Σ'? Show with a Lorentz transformation that both observers agree which rod is marked and where? Explain why this is in agreement to the length contraction in moving reference frames. Hint: To understand the situation, consider drawing spacetime diagrams.