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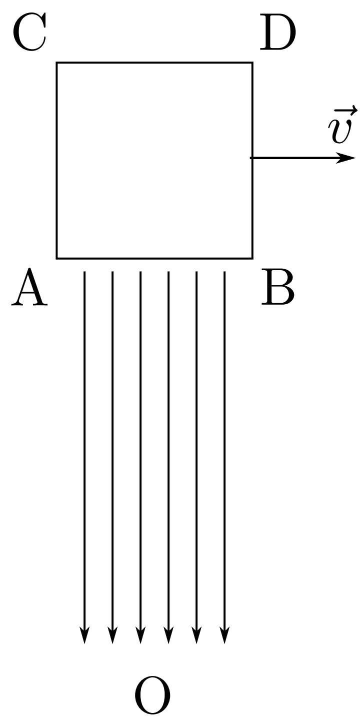
TP 3: Classical Mechanics 2 and Electrodynamics 2 — Sheet 8

Winter Term 2019/2020

Due date: Friday, 13.12.2019, 1pm. Solve the exercises marked with a star* and hand in your solution into the mailbox for TP3. Prepare the other exercises *at home* such that you will be able to present these at the blackboard in the exercise classes of week 50: Tuesday 17.12.2019 or Thursday 19.12.2019.

1. Rotation of a fast cube

Consider a cube of size l that is moving with a relativistic velocity v. From the perspective of an observer (O), two of the faces of the cube are perpendicular to the direction of movement and two others are parallel to the plane defined by the velocity $\vec{v}$ and the observer. Assume that the observer is far away from the cube, i.e. light from the cube can be assumed to be plane (waves).



The observer captures the cube visually by interpreting the light that arrives at the same time at his position. Therefore, light from point C needs to be emitted earlier than that from point A to arrive at the same time at the observer.

a) Due to this, the cube appears to be rotated on such an image. Derive an expression for the angle of rotation of the cube.

b) How would a sphere look alike when moving with relativistic velocity?

2. Lorentz transformation*

By use of suitable Lorentz transformations show the following statements

a) Any light-like vector x^μ with $s^2 = 0$ can be transformed into $(1, 1, 0, 0)^\mu$.

b) Any space-like vector x^μ with $s^2 < 0$ can be transformed into $(0, \sqrt{-\underline{x}^2}, 0, 0)^\mu$.

c) Any time-like vector x^μ with $s^2 > 0$ can be transformed into $(\sqrt{\underline{x}^2}, 0, 0, 0)^\mu$.

Hints: First, consider the transformation $\mathrm{diag}(-1, -1, -1, -1)$ and argue why you can assume without loss of generality $x^0 > 0$ for all three cases. Next, recall the sub-group of the Lorentz transformations that has been discussed in the lecture and explain how it can be used to transform an arbitrary 4-vector (x^0, x^1, x^2, x^3) into a simpler form. Finally, use a suitable Lorentz boost to bring the vectors into the desired form.

3. Transformation of 4-vectors*

a) Consider a transformation between two reference frames Σ and Σ' which move with relative velocity $\vec{v} = v\vec{e}_z$ and transform the 4-velocity $u_1^\mu = \gamma_1(c, \vec{v}_1)$ in reference frame Σ into the reference frame Σ'.

b) Use the result of the previous part and show that the velocity of light in an arbitrary direction is invariant under boosts. (In the lecture this was shown for the propagation direction along the boost direction only.)

c) Assume that we have two intersecting light beams making a non-zero angle ϕ in frame Σ, then show that there always exists another frame of reference Σ'' such that the beams appear to be directed in opposite directions.
Hint: In the lecture, the 4-wave vector k^μ was transformed between reference frames to derive the angles θ and θ' which the corresponding light beams enclose with the axis for the Lorentz boost. Use the result from the lecture to construct the desired frame of reference.

Exercise sheets available at: `http://www.physik.uni-leipzig.de/~kreisel/en/teach.php`