Institut für Theoretische Physik
Universität Leipzig

Dr. A. Kreisel
Rohan Kulkarni

TP 3: Classical Mechanics 2 and Electrodynamics 2 — Sheet 9

Winter Term 2019/2020

Due date: Friday, 20.12.2019, 1pm. Solve the exercises marked with a star* and hand in your solution into the mailbox for TP3. Prepare the other exercises *at home* such that you will be able to present these at the blackboard in the exercise classes of week 2: Tuesday 07.01.2020 or Thursday 09.01.2020.

1. Compton effect*

The Compton effect is due to an elastic collision between a photon and an electron in which the electron acquires kinetic energy, and thus the photon has a smaller kinetic energy, i.e. a larger wavelength than the initial photon. The increase of wavelength depends on the scattering angle θ, the angle between incoming photon and leaving photon.
Verify the Compton condition

$$\lambda' = \lambda + \lambda_c(1 - \cos\theta)$$

where $\lambda_c = \frac{h}{m_e c}$ is the Compton Wavelength of the electron.
Hint: During the elasic collision, the 4-momentum is conserved, i.e.

$$p_1^\mu + p_2^\mu = {p'_1}^\mu + {p'_2}^\mu$$

where p_1^μ and p_2^μ are the 4-momenta before the collision and the primed variables are after the collision. Use a suitable reference frame where the kinetic energy of the electron has a simple expression.

2. Three different tensors

a) Consider the fully antisymmetric tensor of rank 4 $\epsilon^{\alpha\beta\gamma\delta}$ as introduced in the lecture. How many elements of it are different from zero? For the 3 dimensional analog, the tensor ϵ_{ijk}, it holds $\epsilon_{ijk} = \epsilon_{jki} = \epsilon_{kij}$, i.e. the sign is unchanged under cyclic permutations. Does the same property also hold for the antisymmetric tensor of rank 4?

b) Given the energy-momentum tensor $T^{\mu\nu} = \frac{1}{\mu_0}[-F^{\mu\alpha}F^{\nu}{}_{\alpha} + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}]$, where $F^{\mu\nu}$ is the field tensor as introduced in the lecture. Use the Maxwell equations in covariant form to show that this tensor fulfills

$$\partial_\nu T^{\mu\nu} = -F^{\mu\alpha}j_\alpha$$

with j^μ being the 4-current density.

3. Consequences of the Lorentz invariance of $\vec{E} \cdot \vec{B}$ and $B^2 - \frac{E^2}{c^2}$

a) Given the electric field $\vec{E}$ and magnetic induction $\vec{B}$ which enclose the angle ϑ_0 in a certain frame of reference. In which case, i.e. for which angle ϑ_0, is the enclosed angle the same in all reference frames? Explain how you come to this conclusion.

b) Write down the transformation formula for the electric field $\vec{E}$ and the magnetic induction $\vec{B}$ for a Lorentz boost with velocity v in z direction.

c) Now, assume that $\vec{E} \cdot \vec{B}$. Show that it exists a Lorentz transformation which transforms to $\vec{E}' = 0$ if $B^2 - \frac{E^2}{c^2} > 0$ and that it exists a Lorentz transformation which transforms to $\vec{B}' = 0$ if $B^2 - \frac{E^2}{c^2} < 0$. What can you conclude in the case $B^2 - \frac{E^2}{c^2} = 0$?
Hint: Choose a suitable coordinate system where the component of the respective field in the direction of the Lorentz boost is zero.

Exercise sheets available at: `http://www.physik.uni-leipzig.de/~kreisel/en/teach.php`