Institut für Theoretische Physik
Universität Leipzig

Dr. A. Kreisel
Rohan Kulkarni

TP 3: Classical Mechanics 2 and Electrodynamics 2 — Sheet 10

Winter Term 2019/2020

Due date: Friday, 10.01.2020, 1pm. Solve the exercises marked with a star* and hand in your solution into the mailbox for TP3. Prepare the other exercises *at home* such that you will be able to present these at the blackboard in the exercise classes of week 3: Tuesday 14.01.2020 or Thursday 16.01.2020.

1. 4-potential*

Calculate the electric field $\vec{E}$ and the magnetic induction $\vec{B}$ for the following 4-potentials

a) $A^\mu = ax^\mu$, where $a =$ const. and x^μ is the 4-position.

b) $A^\mu = ax^\mu e^{-\lambda x_\nu x^\nu}$, where λ is a constant as well.

c) $A^\mu = a\partial^\mu \theta(X)$, where θ is an arbitrary function of the 4-position X.

d) Find the function $\theta(X)$ that corresponds to the first and second example.

e) Given the fields

$$\vec{E}(r) = \alpha \frac{\vec{r}}{|\vec{r}^3|} \qquad \vec{B} = 0.$$

Find the 4-potential A^μ that describes these fields. Is your choice unique?

2. Energy-momentum tensor*

a) Show that the contraction of the energy momentum tensor $T^{\mu\nu}$ vanishes.

b) Show that the expression $w^2 - \frac{1}{c^2}|\vec{S}|^2$ is a Lorentz invariant. Here $w = \frac{1}{2}(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2)$ is the energy density and $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ is the Poynting vector.
Hint: Rewrite the expression in terms of known Lorentz invariants.

3. Moving electric charges

Consider a particle with electric charge q_1 that creates the potential $\varphi'(\vec{r'}) = \frac{q_1}{4\pi\epsilon_0}\frac{1}{r'}$ and $\vec{A'}(\vec{r'}) = 0$ in a system of reference Σ' where it is at rest.

a) Find the potentials $\varphi(\vec{r})$ and $\vec{A}(\vec{r})$ in a system of reference Σ which moves relative to Σ' with a velocity $\vec{v} = v\vec{e}_z$.

b) Show that $\varphi(\vec{r})$ and $\vec{A}(\vec{r})$ fulfill the condition for Lorentz gauge. Argue in which system of reference you are conveniently showing this and why you can choose your system of reference.

c) Now, explicitly calculate the electric field $\vec{E}(\vec{r})$ and the magnetic induction $\vec{B}(\vec{r})$ in the system of reference Σ. First show explicitly that $\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E}$ and then calculate $\vec{E}$ explicitely. (Note that the desired result for $\vec{B}$ was not derived in the lecture, but the result just stated.)

d) Calculate the force $\vec{F}$ in the system of reference Σ which another particle with charge q_2 is exposed to. The second particle moves with the same velocity than the first particle. Conveniently separate the force $\vec{F}$ into a component parallel and a component perpendicular to the z axis, $\vec{F} = \vec{F}_\perp + \vec{F}_\|$.