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TP 3: Classical Mechanics 2 and Electrodynamics 2 — Sheet 13

Winter Term 2019/2020

Due date: Friday, 31.01.2020, 1pm. Solve the exercises marked with a star* and hand in your solution into the mailbox for TP3. Prepare the other exercises *at home* such that you will be able to present these at the blackboard in the exercise classes of week 6: Tuesday 04.02.2020 or Thursday 06.02.2020.

1. Two disks

Consider two disks of uniform mass distribution. These have a radius R and R' and total mass m and m'. Their motion is restricted in the following way: The first disk can move freely (with the flat side lying down) on a horizontal surface and the second disk (also lying on the flat side) can rotate freely, but has its axis fixed on the first disk at a distance b away from the center of the first disk.
Hint: You can look up the moment of inertia of a disk without further derivation.

a) Write down the constraints of this system, calculate the number of degrees of freedom. Obtain the transformation formula to the generalized coordinates and calculate the generalized velocities.

b) Write down the Lagrange function and the equations of motion.

c) Identify integrals of motion of the system.

d) Calculate the Hamilton function and derive the canonical equations of motion.

2. Relativistic Lagrange function

Given a Lagrange function of a relativistic particle which has two terms $L(\vec{r}, \dot{\vec{r}}) = L_p + L_{em-p}$ where $L_p = \frac{-mc^2}{\gamma(\dot{\vec{r}})}$ is the contribution from the particle with mass m and the relativistic γ factor. $L_{em-p} = q[\vec{A}(\vec{r}) \cdot \dot{\vec{r}} - \varphi(\vec{r})]$ is a term from the presence of electromagnetic fields. Here q is the charge of the particle, $\vec{A}$ is the vector potential and φ is the scalar potential.

a) Derive the following identity: $\vec{a} \times (\vec{\nabla} \times \vec{b}) + \vec{b} \times (\vec{\nabla} \times \vec{a}) = -(\vec{b} \cdot \vec{\nabla})\vec{a} + \vec{\nabla}(\vec{a} \cdot \vec{b}) - (\vec{a} \cdot \vec{\nabla})\vec{b}$

b) Use this relation to derive the equation of motion

$$\frac{d}{dt}(m\gamma\dot{\vec{r}}) = q(\vec{E} + \dot{\vec{r}} \times \vec{B})$$

c) Now, consider the case without electromagnetic fields and calculate the momentum $\vec{p}$ and the energy E of the particle.

Exercise sheets available at: `http://www.physik.uni-leipzig.de/~kreisel/en/teach.php`