Hybrid approach for quantum antiferromagnets in a uniform magnetic field: Application

Effective theory and physical quantities

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Model

- Interacting magnons formulated in terms of hermitian operators ($1/S$ expansion)
  Hasselmann, Kopietz, (2006)

$$\hat{\Psi}_{\mathsf{k}\sigma} = p_\sigma \left[\sqrt{\frac{\nu_{\mathsf{k}\sigma}}{2}} \hat{X}_{\mathsf{k}\sigma} + \frac{i}{\sqrt{2\nu_{\mathsf{k}\sigma}}} \hat{P}_{\mathsf{k}\sigma} \right] \qquad [\hat{X}_{\mathsf{k}\sigma}, \hat{P}_{\mathsf{k}'\sigma'}] = i\delta_{\mathsf{k},-\mathsf{k}'}\delta_{\sigma,\sigma'}$$

$$\hat{H} = E_0^{\mathrm{cl}} + \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4$$

canting ($\hat{H}_1$) — dispersion ($\hat{H}_2$) — interactions due to magnetic field ($\hat{H}_3$)

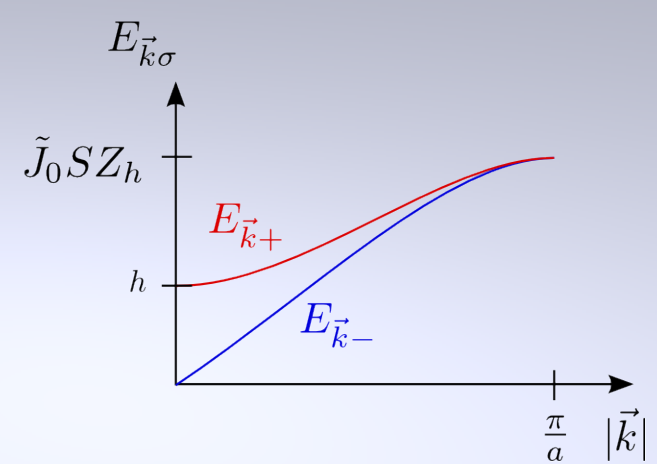
- Physical quantities by evaluation of imaginary time phase space path integral

$$\mathcal{Z} = \int \mathcal{D}[P,X] \exp\left\{ \int_0^\beta d\tau \left[iP\frac{\partial X}{\partial \tau} - H_s(P,X) \right] \right\}$$

Effective Model

- eliminate degrees of freedom associated with the operator of the ferromagnetic fluctuations

$$e^{-S_{\text{eff}}[X_\sigma]} = \int \mathcal{D}[P_\sigma] e^{-S[P_\sigma, X_\sigma]}$$



$$S_{\text{eff}}[X_\sigma] = S_0 + \frac{\beta}{2} \sum_{K\sigma} \frac{E^2_{\mathsf{k}\sigma} + \omega^2}{\Delta_{\mathsf{k}\sigma}} X_{-K\sigma} X_{K\sigma}$$

$$+\beta \sqrt{\frac{2}{N}} \sum_{K_1 K_2 K_3} \delta_{K_1+K_2+K_3,0} \Big[\frac{1}{3!} \Gamma^{(3)}_{---}(K_1, K_2, K_3) X_{K_1-} X_{K_2-} X_{K_3-}$$

$$+ \frac{1}{2!} \Gamma^{(3)}_{-++}(K_1; K_2, K_3) X_{K_1-} X_{K_2+} X_{K_3+} \Big]$$

$$K = (\mathsf{k}, i\omega_n)$$

Hybrid approach: Comparison

- generalization of Non-Linear-Sigma-Model for QAF subject to magnetic field

$$S_{\mathrm{NLSM}}[\Omega] \approx -\beta V \frac{\chi}{2} h^2 + \frac{\chi}{2} \int_K \sum_\sigma (\omega^2 + c^2 \mathsf{k}^2 + m_\sigma^2) \Pi_{-K\sigma} \Pi_{K\sigma} \qquad \chi = \frac{\rho}{c^2}$$

$$-i\chi h \int_0^\beta d\tau \int d^D r \Pi_+^2 \partial_\tau \Pi_- + \mathcal{O}(\Pi_\sigma^4)$$

$$\partial_\tau \Omega \to (\partial_\tau - i\mathsf{h}\times)\Omega$$

$$X_\sigma \hat{=} \Pi_\sigma$$

$$E_{\mathsf{k}+}^2 \approx m\Delta_0 h + c_+^2 \mathsf{k}^2$$

$$E_{\mathsf{k}-}^2 \approx c_-^2 \mathsf{k}^2$$

$$S_{\mathrm{eff}}[X_\sigma] = S_0 + \frac{\beta}{2} \sum_{K\sigma} \frac{E_{\mathsf{k}\sigma}^2 + \omega^2}{\Delta_{\mathsf{k}\sigma}} X_{-K\sigma} X_{K\sigma} +$$

$$+\beta\sqrt{\frac{2}{N}} \sum \left[\frac{1}{3!}\Gamma^{(3)}_{---} X_- X_- X_- + \frac{1}{2!}\Gamma^{(3)}_{-++} X_- X_+ X_+\right]$$

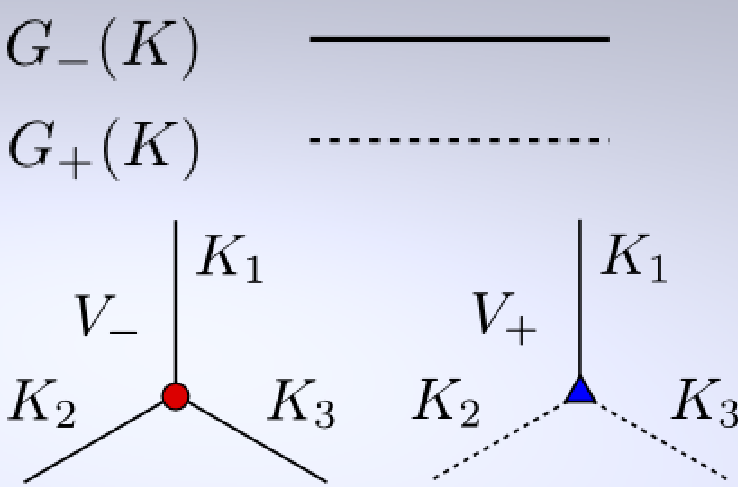
$$\Gamma^{(3)}_{---} \approx 0 \qquad \Gamma^{(3)}_{-++} \approx -2\frac{\lambda}{\sqrt{8S}}\omega_1$$

$1/S$ Corrections

- diagrammatic perturbation theory

$$S^{\text{int}}_{\text{eff}}[X_\sigma] \quad = \beta\sqrt{\tfrac{2}{N}}\sum\left[\tfrac{1}{3!}V_-^{(3)}X_-X_-X_- + \tfrac{1}{2!}V_+^{(3)}X_-X_+X_+\right]$$

$$G_\sigma(K) = \frac{\Delta_{\mathsf{k}\sigma}}{E^2_{\mathsf{k}\sigma}+\omega^2}$$

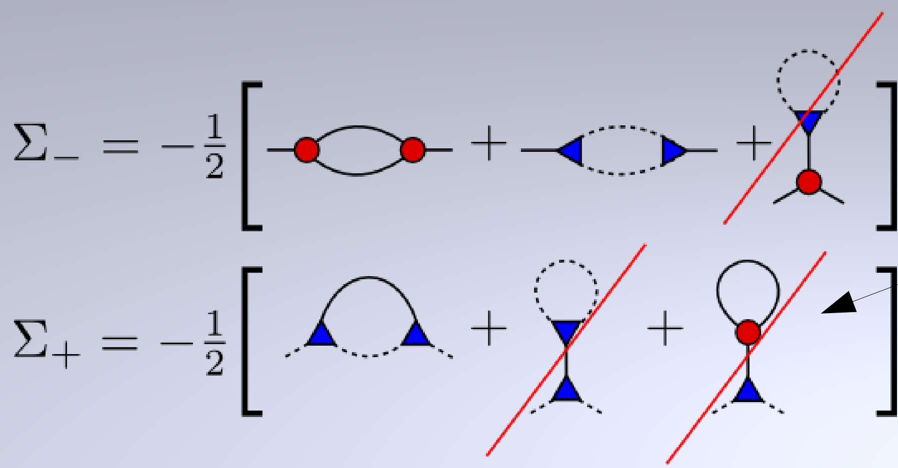


$$V_+ = \frac{\Delta_0\lambda}{\sqrt{8S}}\Big[(\gamma_{\mathsf{k}_1}-\gamma_{\mathsf{k}_2}-\gamma_{\mathsf{k}_3})\frac{\omega_1}{\Delta_{\mathsf{k}_1-}} + \gamma_{\mathsf{k}_2}\frac{\omega_3}{\Delta_{\mathsf{k}_3+}} + \gamma_{\mathsf{k}_3}\frac{\omega_2}{\Delta_{\mathsf{k}_2+}}\Big]$$

$$V_- = \frac{\Delta_0\lambda}{\sqrt{8S}}\Big[\gamma_{\mathsf{k}_1}\frac{\omega_1}{\Delta_{\mathsf{k}_1-}} + \gamma_{\mathsf{k}_2}\frac{\omega_2}{\Delta_{\mathsf{k}_2-}} + \gamma_{\mathsf{k}_3}\frac{\omega_3}{\Delta_{\mathsf{k}_3-}}\Big]$$

Self energy

- perturbation theory: $1/S$ corrections



no frequency dependence, negligible

$$\Sigma_-(K) = \frac{1}{\beta N}\sum_{K'}\sum_{\sigma} G_\sigma(K')G_\sigma(K'+K)V_\sigma^2(K,K',-K-K')$$

$$\Sigma_+(K) = \frac{1}{\beta N}\sum_{K'} G_-(K')G_+(K'+K)V_+^2(K',K,-K-K')$$

Results

- leading order expansion of self energy

$$\Sigma_-(K) = C^\omega \omega^2 + C^k c_0^2 \mathsf{k}^2 + \mathcal{O}(\omega^4, k^4)$$

Zhitomirsky, Nikuni, 1998

- full propagator

$$G_-(K) = \frac{\Delta_{\mathsf{k}-}}{\omega^2 + E_{\mathsf{k}-}^2 + \Delta_{\mathsf{k}-}\Sigma_-(K)} \approx \frac{Z_-\Delta_0 n^2}{\omega^2 + c_-(h)^2\mathsf{k}^2}$$

non analytic in h^2

- spin wave velocity of gapless mode

$$\frac{c_-^2}{c_0^2} \approx 1 - \Delta_0 n^2 C^\omega \approx 1 - \frac{6\sqrt{3}\,\tilde{h}^2}{\pi^2 S} \ln\left(\frac{2}{\tilde{h}}\right) \qquad D = 3$$

$$\frac{c_-^2}{c_0^2} \approx 1 - \Delta_0 n^2 C^\omega \approx 1 - \frac{2\tilde{h}}{\pi S} \qquad D = 2$$

$$\tilde{h} = \frac{h}{\Delta_0}$$

Conclusion

- effective model for QAF subject to magnetic field
- perturbation theory for relevant degrees of freedom
- calculation of non analytic corrections to spin wave velocity