Time-dependent spin-wave theory

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1.1 Model

- Heisenberg ferromagnet

\[ \mathcal{H}(t) = -\frac{1}{2} \sum_{i,j} J_{ij} S_i \cdot S_j - \sum_i h_i(t) \cdot S_i, \]

- time-dependent magnetic field (rotating magnet)

\[ h(t) = \begin{pmatrix} h_\perp \cos(\omega t) \\ -h_\perp \sin(\omega t) \\ h_z \end{pmatrix} \]

- Goal: generalization of spin wave-approach to magnet in time-dependent magnetic field
2.1 Spin-wave approach: adiabatic approximation

- static minimization of the classical ground-state energy (as in time-independent case)
- projection of spin operators onto time-dependent basis
- Holstein Primakoff transformation Holstein, Primakoff (1940)
- local magnetization follows the magnetic field adiabatically

\[ M_{\text{ad}}(t) = \left[ S - \frac{1}{N} \sum_k \frac{1}{e^{\beta(\epsilon_k + h)} - 1} \right] \begin{pmatrix} \cos \theta_0 \cos(\omega t) \\ - \cos \theta_0 \sin(\omega t) \\ \sin \theta_0 \end{pmatrix} \]

- \( \epsilon_k = S(J_0 - J_k) \) 
- \[ \cos \theta_0 = \frac{h_z}{\sqrt{h_\perp^2 + h_z^2}} \]
2.2 Spin-wave approach: Perturbation theory in $h_\perp$

- **split contributions to the magnetic field**
  \[
  h(t) = \begin{pmatrix} h_\perp \cos(\omega t) \\ -h_\perp \sin(\omega t) \\ h_z \end{pmatrix} = h_0 + h_1(t)
  \]

- **expand in laboratory frame**
  \[
  \mathcal{H}_z \approx -DNJS^2 - Nh_z S + \sum_k (\epsilon_k + h_z) b_k^\dagger b_k
  \]
  \[
  \mathcal{V}(t) \approx -\frac{h_\perp}{2} \sqrt{2S\sqrt{N}} \left[ e^{i\omega t} b_{k=0} + e^{-i\omega t} b_{k=0}^\dagger \right] \propto h_\perp
  \]

- **calculate magnetization by solving the set of Heisenberg equations of motion**
  \[
  i \frac{dA}{dt} = [A, H]
  \]

- **result**
  \[
  M_{\text{lab}}(t) = \frac{h_\perp S}{h_z - \omega} \left[ \cos(\omega t) x - \sin(\omega t) y \right] + M_{h_z} z
  \]
  unphysical divergence for $h_z \to \omega$
3.1 Construction of proper basis

- idea: factorize unitary time-evolution operator
  \[ i\hbar \frac{dU(t)}{dt} = H(t)U(t) \quad U(t) = U_0(t) \tilde{U}(t) \]
- choose trivial part to rotate system to the direction of the true magnetization
  \[ U_0(t) = e^{-i \sum_i \alpha_i(t) \cdot S_i} \quad \alpha_i(t) \propto m_i(t) \]
- new operator equation determines the dynamics
  \[ i\hbar \frac{d\tilde{U}(t)}{dt} = \tilde{H}(t)\tilde{U}(t) \]
- effective Hamiltonian
  \[ \tilde{H}(t) = U_0^\dagger(t)H(t)U_0(t) + \tilde{H}_B(t) \]

Hamiltonian from adiabatic limit

\[ \tilde{H}_B(t) = -\sum_i \left[ \omega_i^{(1)}(t)\tilde{S}_i^{(1)} + \omega_i^{(2)}(t)\tilde{S}_i^{(2)} + \omega_{i\parallel}(t)\tilde{S}_i^{\parallel} \right] = -iU_0^\dagger(t)\partial_t U_0(t) \]

Lin, Commun. Theor. Phys. (‘05)
3.2 Results: Spin-wave theory in proper basis

- transform spin operator
  \[ \tilde{S}_i(t) = e^{i\alpha_i(t)}S_i e^{-i\alpha_i(t)} = e^{\alpha_i(t)} \times S_i \]
- apply spin-wave theory as expansion around true classical ground state
- result for magnetization
  \[ M(t) = \left[ S - \frac{1}{N} \sum_k \frac{1}{e^{\beta(\epsilon_k + \tilde{h})} - 1} \right] \begin{pmatrix} \cos \theta \cos(\omega t) \\ - \cos \theta \sin(\omega t) \\ \sin \theta \end{pmatrix} \]

\[
\frac{\tilde{M}_\omega}{S} = \begin{cases} 
\text{adiabatic approximation} & \text{if} \ \omega \ll h_z \\
\text{proper rotating frame} & \text{if} \ \omega \approx h_z \\
\text{lab. basis} & \text{if} \ \omega \gg h_z 
\end{cases}
\]

\[
\cos \theta = \frac{(h_z - \omega)}{\sqrt{h^2 + (h_z - \omega)^2}}
\]

Effective magnetic field
\[ h_z \rightarrow \tilde{h}_\omega = h_z - \omega \]
4.1 Application to pumped ferromagnet

- Model: ferromagnet with rotating single-ion anisotropy (mimics dipole-dipole interactions)

\[ \mathcal{H}(t) = -\frac{1}{2} \sum_{ij} J_{ij} S_i \cdot S_j - h \sum_i S_i^z \]
\[ - \frac{A}{2} \sum_i \left\{ [S_i \cdot n(t)]^2 - [S_i \cdot (z \times n(t))]^2 \right\} \]

- construct proper rotating basis

- use linear spin-wave theory (without interactions between magnons) to calculate the spectrum and magnetization
4.2 Results

- **spectrum**

\[ E_k = \sqrt{(\epsilon_k + h - \omega)^2 - h_c^2} \]

\[ |h - \omega| > AS = h_c \]

\[ E_k = \sqrt{\left[ \epsilon_k + \frac{3h_c}{2} - \frac{(h - \omega)^2}{2h_c} \right]^2 - \left[ \frac{h_c}{2} + \frac{(h - \omega)^2}{2h_c} \right]^2} \]

\[ |h - \omega| < AS \]

- **magnetization**

\[ M(t) = \tilde{M}_\omega m_\omega(t) \]

\[ m_\omega(t) = \sin \theta [\cos(\omega t) \times - \sin(\omega t) y] + \cos \theta z \]

frequency dependent canting angle

\[ \tilde{M}_\omega = S + \frac{1}{2} - \frac{1}{N} \sum_k \frac{1}{E_k} \left[ \epsilon_k + \frac{3h_c}{2} - \frac{(h - \omega)^2}{2h_c} \right] \]

\[ \times \left[ \frac{1}{e^{\beta E_k} - 1} + \frac{1}{2} \right] \]
5.1 Summary

- Conventional approaches fail:
  - adiabatic approximation for nonzero frequencies
  - perturbation theory for large effective field
- Construction of proper rotating basis by factorization of time-evolution operator allows to use linear spin-wave theory.
- Testable predictions for pumped ferromagnet for example on thin films of YIG

Demokritov, et al., Nature, 06