# PHY2049 - Fall 2016 - HW6 Solutions

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These are solutions to Halliday, Resnick, Walker Chapter 28, No: 10, 16, 30, 35, 38, 39, 51, 65

## 1 28.10

**A proton travels through uniform magnetic and electric fields. The magnetic field is $\vec{B} = (-2.50\ \textbf{mT}\ )\,\hat{i}$. At one instant the velocity of the proton is $v = (2000\ \textbf{m/s}\ )\,\hat{j}$. At that instant and in unit-vector notation, what is the net force acting on the proton if the electric field is (a) $(4.00\textbf{V/m})\,\hat{k}$ (b) $(-4.00\ \textbf{V/m}\ )\,\hat{k}$, and (c) $(4.00\textbf{V/m})\,\hat{i}$ ?**

The Lorentz force invloving both fields is

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

The proton charge is just $e$. In all cases $\vec{v} =$ 2000 m/s $\hat{j} = v_y\hat{j}$ and $\vec{B} =$ -2.50 mT $\hat{i} = B_x\hat{i}$. For case (a) the electric field is $\vec{E} = 4.00\ \hat{k} = E_z\hat{k}$

$$\vec{F} = e\left(\begin{bmatrix} 0 \\ 0 \\ E_z \end{bmatrix} + \begin{bmatrix} 0 \\ v_y \\ 0 \end{bmatrix} \times \begin{bmatrix} B_x \\ 0 \\ 0 \end{bmatrix}\right)$$

You should evaluate the cross product first

$$\begin{bmatrix} 0 \\ v_y \\ 0 \end{bmatrix} \times \begin{bmatrix} B_x \\ 0 \\ 0 \end{bmatrix} = v_y B_x \left(\hat{y} \times \hat{x}\right) = v_y B_x \left(-\hat{z}\right) = \begin{bmatrix} 0 \\ 0 \\ -v_y B_x \end{bmatrix}$$

Pluggign this into the equations for the Lorentz force we find

$$\vec{F} = e\left(\begin{bmatrix} 0 \\ 0 \\ E_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -v_y B_x \end{bmatrix}\right)$$

$$\vec{F} = e\left(E_z - v_y B_x\right)\hat{z}$$

$$\vec{F} = 1.602\text{x}10^{-19}\text{C}\left(4\ \text{V/m} - 2000\ \text{m/s}\left(\text{-}2.5\text{x}10^{-3}\text{T}\right)\right) = 1.44\text{x}10^{-18}\ \text{N}\ \hat{k}$$

For (b) and (c), simply evalue this expression for different values of $\vec{E}$. Note that in part (c) $\vec{E}$ is in the x-direction (not z) so you'll end up with an x-component as well as a z-component of the resulting force.

# 2 28.16

**Figure 28-34 shows a metallic block, with its faces parallel to coordinate axes. The block is in a uniform magnetic field of magnitude 0.020 T. One edge length of the block is 25 cm; the block is not drawn to scale. The block is moved at 3.0 m/s parallel to each axis, in turn, and the resulting potential difference V that appears across the block is measured. With the motion parallel to the y-axis, V = 12 mV; with the motion parallel to the z axis, V = 18 mV; with the motion parallel to the x axis, V = 0.What are the block lengths (a) $d_x$, (b) $d_y$, and (c) $d_z$?**

We make use of the Hall Effect, in which we require that the strength of force due to the electric field is equal to that of the magnetic field

$$\left|e\vec{E}\right| = e\left|\vec{v} \times \vec{B}\right|$$

$$|E| = |v|\,|B|$$

The hall potential difference is

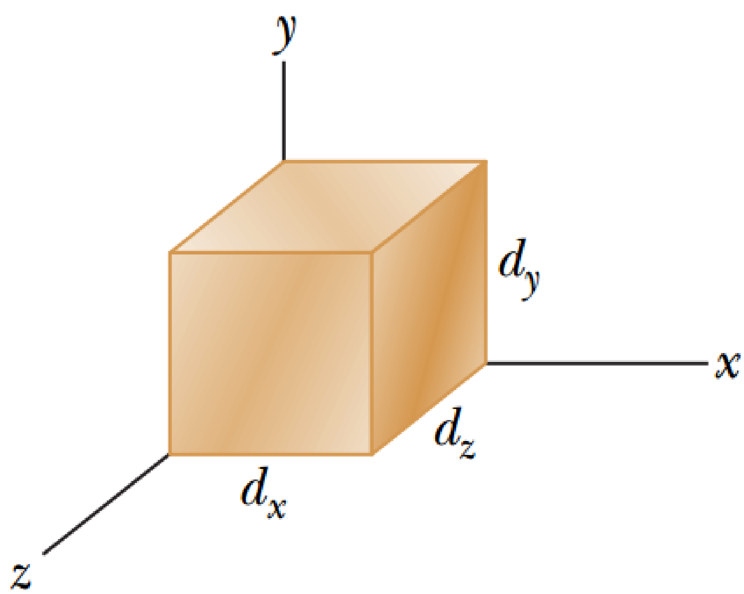


**Fig. 28-34**
Problems 15 and 16.

Figure 1: Problem 28.16

$$V = Ed$$

Replace $E$ with $vB$ and find

$$V = vBd$$
$$\frac{V}{vB} = d$$

$$d_z = \frac{0.012\text{ V}}{(3.0\text{ m/s})\,(0.02\text{ T })} = 0.02\text{m}$$
$$d_y = \frac{0.018\text{ V}}{(3.0\text{ m/s})\,(0.02\text{ T })} = 0.03\text{m}$$

Then $d_x$ must be 0.025 m by process of elimination.

# 3 28.30

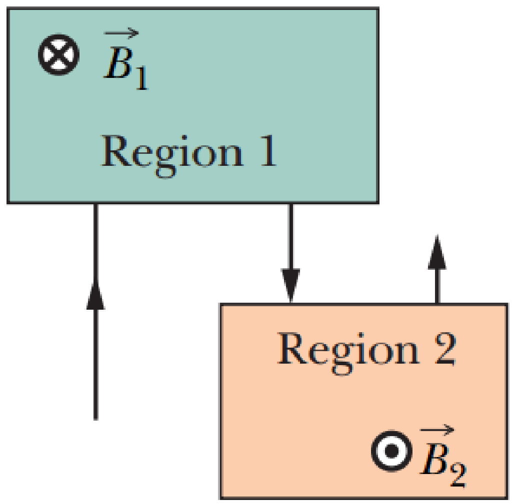


**Fig. 28-39**
Problem 30.

Figure 2: Problem 30

**In Fig. 28-39, an electron with an initial kinetic energy of 4.0 keV enters region 1 at time $t = 0$. That region contains a uniform magnetic field directed into the page, with magnitude 0.010 T. The electron goes through a half-circle and then exits region 1, headed toward region 2 across a gap of 25.0 cm. There is an electric potential difference $\Delta V = 2000$ V across the gap, with a polarity such that the electrons speed increases uniformly as it traverses the gap. Region 2 contains a uniform magnetic field directed out of the page, with magnitude 0.020 T. The electron goes through a half-circle and then leaves region 2. At what time $t$ does it leave?**

If the electron spends an amount of time $t_1$ in region 1, $t_{gap}$ in the gap, and $t_2$ in region two, the total time of this journey is

$$t_{\text{total}} = t_1 + t_{gap} + t_2$$

In regions 1 and 2, the electron makes a half circle

$$t = \frac{d}{v} = \frac{\pi R}{v}$$

In circular motion due to a magnetic field, we have

$$m\frac{v^2}{R} = qvB$$

so $v = qBR/m$ and the $t$ becomes

$$t = \frac{d}{v} = \frac{\pi R}{qBR/m} = \frac{\pi m}{qB}$$

We can use this to get $t_1$ and $t_2$:

$$t_1 = \frac{\pi m_e}{eB_1} = \frac{\pi \text{ 9.109e-31 kg}}{\text{1.602e-19 C} \ \times \ \text{0.01 T}} = 1.786 \text{ ns}$$

$$t_2 = \frac{\pi m_e}{eB_2} = \frac{\pi \text{ 9.109e-31 kg}}{\text{1.602e-19 C} \ \times \ \text{0.02 T}} = 0.893 \text{ ns}$$

The time in the gap requires us to do a little work. The electron accelerates trough a potential difference of 2 kV. So it gaines 2 keV of energy. It enters the gap with $E_i = 4$ keV and thus leaves with $E_f = 6$ keV. It is useful here to know 1 keV = 1.602e-16 J. We express speed in terms of E using $v = \frac{1}{2}mv^2$ and find

$$v_i = \sqrt{\frac{2E_i}{m_e}} = 3.7509\text{e}7 \text{ m/s}$$

$$v_f = \sqrt{\frac{2E_f}{m_e}} = 4.5939\text{e}7 \text{ m/s}$$

We can use the kinematic equation

$$\frac{1}{2}\left(v_f + v_i\right) t_{gap} = d$$

and use d = 25e-2 m to obtain

$$t_{gap} = 5.99\text{e-}9 \text{ s} = 5.99 \text{ ns}$$

Therefore the total transit time is

$$t_{total} = t_1 + t_{gap} + t_2 = 1.786 \text{ ns} + 5.99 \text{ ns} + 0.893 \text{ ns} = 8.67 \text{ ns}$$

# 4 28.35

**A proton circulates in a cyclotron, beginning approximately at rest at the center. Whenever it passes through the gap between dees, the electric potential difference between the dees is 200 V. (a) By how much does its kinetic energy increase with each passage through the gap? (b) What is its kinetic energy as it completes 100 passes through the gap? Let $r_{100}$ be the radius of the protons circular path as it completes those 100 passes and enters a dee, and let $r_{101}$ be its next radius, as it enters a dee the next time. (c) By what percentage does the radius increase when it changes from $r_{100}$ to $r_{101}$? That is, what is**

$$\textbf{percent increase} = \frac{r_{101} - r_{100}}{r_{100}}$$

(a) The increase in kinetric energy $E$ will be 200 eV. That's why we call them electron volts. To see this explicitly,

$$\Delta\ E = q\Delta V = e \times\ 200\text{V}$$

because the proton's charge is $e$.

(b) It's kinetic energy after 100 passes will be

$$100e\Delta V = 100e\,(200 \text{ V}) = 20.0 \text{ keV}$$

Note that for part (c) we will want to recognize that after 101 passes, the energy will be

$$101e\Delta V = 100e\,(200 \text{ V}) = 20.2 \text{ keV}$$

(c) Whenever asked about cyclotrons, synchrotrons, or any circulating particles in a magnetic field, just start with

$$m\frac{v^2}{r} = qvB$$

Get $r$ alone:

$$r = \frac{mv}{eB}$$

We don't have $v$ but we have kinetic energy $E$ which is related by

$$E = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E}{m}}$$

so

$$r = \frac{m}{eB}\sqrt{\frac{2E}{m}} = \frac{\sqrt{2mE}}{eB}$$

now

$$r_{100} = \frac{\sqrt{2m(\ 20.0\ \mathrm{keV}\ )}}{eB}$$

$$r_{101} = \frac{\sqrt{2m(\ 20.2\ \mathrm{keV}\ )}}{eB}$$

$$\frac{r_{101} - r_{100}}{r_{100}} = \frac{\sqrt{20.2} - \sqrt{20}}{\sqrt{20}} = 0.5\%$$

# 5 28.38

**In a certain cyclotron a proton moves in a circle of radius 0.500 m. The magnitude of the magnetic field is 1.20 T. (a) What is the oscillator frequency? (b) What is the kinetic energy of the proton, in electron-volts?**

The knee-jerk reaction by now should be to write

$$m\frac{v^2}{r} = qvB$$

$$mv = qBr$$

To get frequency, lets think about time first. The time should be $t = d/v$. First lets get $v$

$$mv = qBr$$

$$v = \frac{qBr}{m}$$

The the time is

$$t = \frac{d}{v} = \frac{2\pi r}{qBr/m} = \frac{2\pi m}{qB}$$

The frequency $f = 1/t$. Use $q = e$ for a proton, and $m = m_p =$ 1.6726219e-27 kg

$$f = \frac{eB}{2\pi m_p} = \text{ 18.294 MHz}$$

The kinetic energy $E$ is

$$E = \frac{1}{2}mv^2$$

where

$$v = \frac{qBr}{m}$$

Thus,

$$E = \frac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \text{ 2.762e-12 J}$$

Recalling **1 eV = 1.602e-19 J** we find the kinetic energy is **17.24 MeV.**

# 6 28.39

**A horizontal power line carries a current of 5000 A from south to north. Earths magnetic field (60.0 $\mu$T) is directed toward the north and inclined downward at 70.0° to the horizontal. Find the**

**(a) magnitude and (b) direction of the magnetic force on 100 m of the line due to Earths field.**

The force exerted by an external magnetic field $\vec{B}$ on a wire of differential length $d\ell$ carrying current $i$ in the direction of $\vec{\ell}$ is

$$d\vec{F} = i d\vec{\ell} \times \vec{B}$$

We need to do nothing more than apply this formula exactly. Assume the earth is flat and the x-y plane lies in the plane of the earth. Let the $+\hat{y}$ direction be North. Then the current direction is $+\hat{y}$. This is the direction of $d\ell$ in the formula above. We have a 100m line so

$$\ell = (100 \text{ m}) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

The direction of $\vec{B}$ is given to us: it's in the $y - z$ plane at 70.0° with respect to north, which is $+\hat{y}$.

By the way, if you do not draw this on paper, you will never be able to get the components of $\vec{B}$ correct and correctly apply the formulas. **Always draw. If you don't, you will mess up and get it wrong.**

Anyway, we now know $\vec{B}$ ... with a magnitude given as $\left|\vec{B}\right| = 60.0\mu\text{T}$

$$\vec{B} = \left|\vec{B}\right| \begin{bmatrix} 0 \\ \cos(\theta) \\ -\sin(\theta) \end{bmatrix}$$

$$\vec{B} = (60\ \mu\text{T}) \begin{bmatrix} 0 \\ 0.342 \\ \text{-}0.939 \end{bmatrix}$$

so with $i =$ 5000A

$$\vec{F} = i\left(\vec{\ell} \times \vec{B}\right)$$

$$\vec{F} = (5000\text{A})\,(100\text{m})\,(60\mu\text{T}) \left( \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0.342 \\ \text{-}0.939 \end{bmatrix} \right)$$

$$\vec{F} = (5000\text{A})\,(100\text{m})\,(60\mu\text{T})\left(\text{-}0.939\hat{x}\right)$$

$$\vec{F} = \text{-28.17 N } \hat{x}$$

In our coordinate system, north was $+\hat{y}$, so $-\hat{x}$ is west.

# 7 28.51

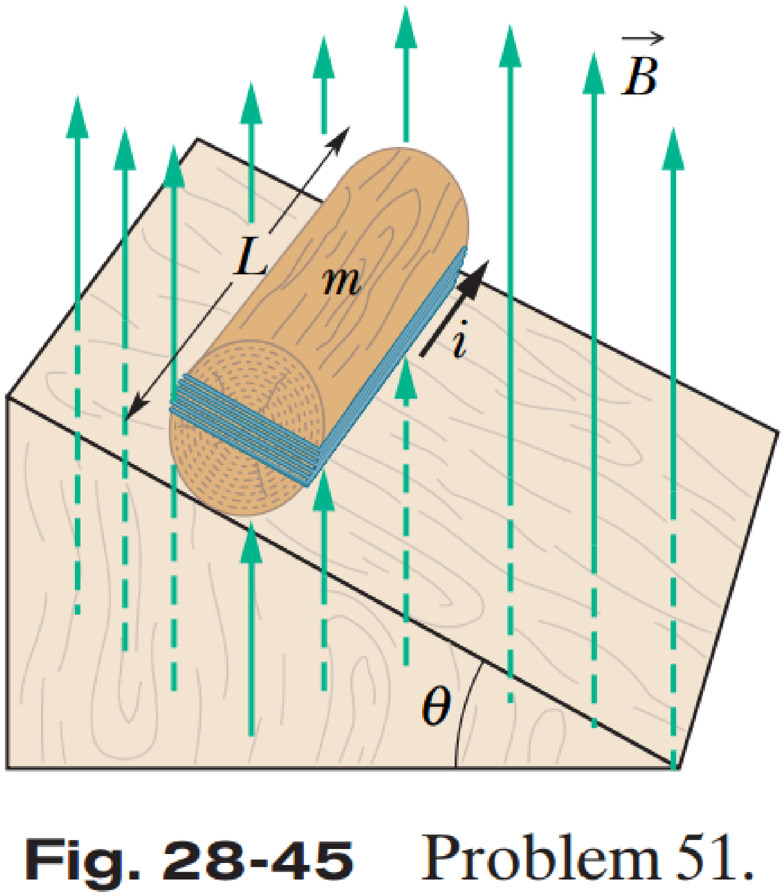


Figure 3: Problem 51

**Figure 28-45 shows a wood cylinder of mass** $m = 0.250$ **kg and**

**length $L = 0.100$ m , with $N = 10.0$ turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle $\theta$ to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude $0.500$ T, what is the least current $i$ through the coil that keeps the cylinder from rolling down the plane?**

The torque due to an external $\vec{B}$ on a current loop is

$$\vec{\tau_B} = \vec{\mu} \times \vec{B}$$

where $\mu$ is the magnetic moment of the loop, defined as

$$|\vec{\mu}| = NiA$$

where $A$ is the area of the loop, $i$ is the current, and $N$ is the number of turns. For our log, the area is the length times twice the radius of the log

$$A = 2rL$$

so

$$|\vec{\mu}| = 2NirL$$

The magnitude of the torque due to $\vec{B}$ is then just

$$|\vec{\tau_B}| = \left|\vec{\mu} \times \vec{B}\right| = |\vec{\mu}| \left|\vec{B}\right| \sin\theta$$

$$|\vec{\tau_B}| = 2NirLB \sin\theta$$

To keep the log from rolling, we need this to be equal to the torque due to gravity

$$|\vec{\tau_g}| = rmg \sin\theta$$

So we set the torques equal

$$\tau_B = \tau_g$$

$$2NirLB \sin\theta = rmg \sin\theta$$

$$i = \frac{mg}{2NLB} = \text{ 2.45 A}$$

# 8 28.65

**A wire of length 25.0 cm carrying a current of 4.51 mA is to be formed into a circular coil and placed in a uniform magnetic field $\vec{B}$ of magnitude 5.71 mT. If the torque on the coil from the field is maximized, what are (a) the angle between and the coils magnetic dipole moment and (b) the number of turns in the coil? (c) What is the magnitude of that maximum torque?**

The torque is given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Obviously, the angle for maximum torque is when $\theta = \text{ 90 deg }$, because the cross product is maximal then (and vanishes when $\theta = 0$.

Now we must optimize $\vec{\mu}$.

$$\vec{\mu} = NiA$$

The length $L$ is the circumference for $N = 1$, so $L = 2\pi r$. We know $A\pi r^2$, so you can hopefully show that a single-turn loop of string with length $L$ is

$$A = \frac{L^2}{4\pi}$$

If there are more turns, the effective $L$ becomes $L/N$

$$A = \frac{\left(\frac{L}{N}\right)^2}{4\pi}$$

and the corresponding $\mu$ is

$$\mu = NiA = Ni\frac{\left(\frac{L}{N}\right)^2}{4\pi} = \frac{iL^2}{4\pi N}$$

So $\mu$ is maximized when $N = 1$. Now it only remains to calculate the magnitude of torque in these conditions; $\theta = \text{ 90 deg}$ and $N = 1$:

$$|\vec{\tau}| = \frac{iL^2}{4\pi}B$$

$$\tau = \frac{(4.51\text{e-}3\text{ A})\,(0.25\text{ m})^2}{4\pi}\,(5.71\text{e-}3\text{ T}) = \;1.28\text{e-}7\text{ N}\cdot\text{ m}$$