PHY2049 - Fall 2016 - HW7 Solutions

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These are solutions to Halliday, Resnick, Walker Chapter 29 No: 18, 25, 30, 39, 41, 48, 55, 62

1 29.18

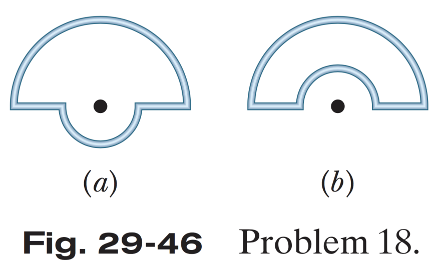

Figure 1: Problem 29.18

A current is set up in a wire loop consisting of a semicircle of radius 4.00 cm, a smaller concentric semicircle, and two radial straight lengths, all in the same plane. Figure 29-46a shows the arrangement but is not drawn to scale. The magnitude of the magnetic field produced at the center of curvature is 47.25 μT. The smaller semicircle is then flipped over (rotated) until the loop is again entirely in the same plane (Fig. 29-46b). The magnetic

field produced at the (same) center of curvature now has magnitude 15.75 μT, and its direction is reversed. What is the radius of the smaller semicircle?

The radial sections contribute nothing to the field at the center - only the arcs contribute.

In configuration (a), the fields at the center due to each semicircle point in the same direction and are thus additive.

In configuration (b), the fields oppose eachother so they subtract.

Let the large radius and smaller unknown radius be $R = 4.00$cm and r respectively. Let also the magnitudes of the fields produced by each semicirlce be B_R and B_r. We have

$$47.25\mu\text{T} = B_R + B_r$$
$$15.75\mu\text{T} = B_R - B_r$$

We know the general form for the $\vec{B}$ due to a circular arc of radius R subtending angle ϕ

$$\left|\vec{B}\right| = \frac{\mu_0 i \phi}{4\pi R}$$

We can now express B_R and B_r using $\phi = \pi$ for half circles

$$47.25\mu\text{T} = B_R + B_r = \frac{\mu_0 i}{4}\left(\frac{1}{R} + \frac{1}{r}\right)$$
$$15.75\mu\text{T} = B_R - B_r = \frac{\mu_0 i}{4}\left(\frac{1}{R} - \frac{1}{r}\right)$$

rearranging we find

$$47.25\mu\text{T}\frac{4}{\mu_0 i} = \left(\frac{1}{R} + \frac{1}{r}\right)$$
$$15.75\mu\text{T}\frac{4}{\mu_0 i} = \left(\frac{1}{R} - \frac{1}{r}\right)$$

We don't know the current i, so let's get rid of it but taking the ratio of these equations. The left side becomes 3:

$$\frac{47.25\mu\text{T}}{15.75\mu\text{T}} = \frac{\frac{1}{R} + \frac{1}{r}}{\frac{1}{R} - \frac{1}{r}}$$

$$3 = \frac{\frac{1}{R} + \frac{1}{r}}{\frac{1}{R} - \frac{1}{r}}$$

$$\frac{3}{R} - \frac{3}{r} = \frac{1}{R} + \frac{1}{r}$$

$$\frac{2}{R} = \frac{4}{r}$$

$$2r = 4R$$

$$r = \frac{R}{2} = \frac{4.00\text{cm}}{2} = 2.00\text{cm}$$

2 29.25

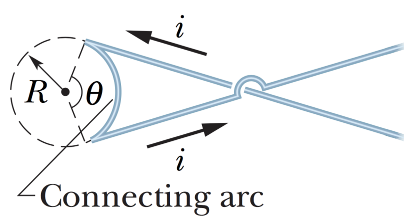

Fig. 29-52
Problem 25.

Figure 2: Problem 28.25

A wire with current $i = 3.00$A is shown in Fig. 29-52. Two semi-infinite straight sections, both tangent to the same circle, are connected by a circular arc that has a central angle θ and runs along the circumference of the circle. The arc and the two straight sections all lie in the same plane. If $B = 0$ at the circles center, what isθ?

The field due to both straight wires is out of the page while the field due to the circular arc is into the page. We just have to make them cancel.

$$\text{(2 semi-infinite wire fields)} = \text{(field due to a circular arc)}$$

If you forgot the field due to a semi-infinite wire, you're not alone. So did I. But I know I can arrive at the expression from the precious few things I do know cold, such as Ampere's law. Let's do the work of getting that expression now using Ampere's law and some symmetry.

Your book derives the field due to a semi-infinite straight wire using the Biot-Savart law. This is formal, but probably the worst way to do it by hand quickly. You should avoid using Biot-Savart at all costs, in favor of Ampere's law, when possible.

The field due to a semi-infinite wire is half that of the infinite wire. That case is easy to handle with Ampere's law:

$$\oint_{loop} \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

In the case of the **infinite wire**, Ampere's law delivers

$$B_{inf}\ 2\ \pi\ R = \mu_0 i$$

$$B_{inf} = \frac{\mu_0 i}{2\pi R}$$

Now $\vec{B}$ for the semi-infinite case has the same direction and half the magnitude

$$B_{semi-inf} = \frac{\mu_0 i}{4\pi R}$$

This is the B we will use for the wires. Remember

$$2B_{semi-inf} = B_{arc}$$

The B due to a circular arc with radius R subtending an angle θ is still

$$B_{arc} = \frac{\mu_0 i\theta}{4\pi R}$$

Now we just enforce $2B_{semi-inf} = B_{arc}$

$$2\frac{\mu_0 i}{4\pi R} = \frac{\mu_0 i\theta}{4\pi R}$$

$$2 = \theta$$

The angle here is measured in radians. $\theta = 2\text{rad} = 114.592°$.

3 29.30

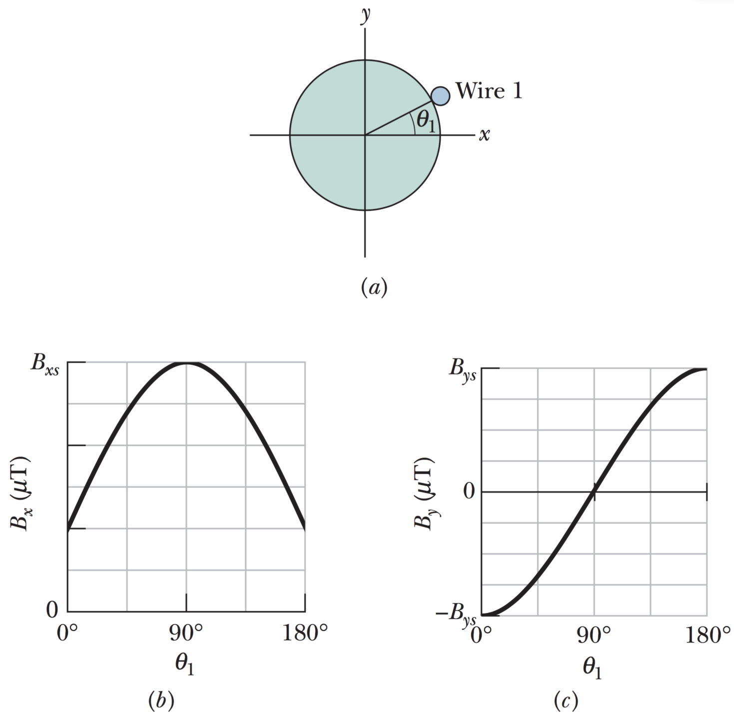

Fig. 29-57 Problem 30.

Figure 3: Problem 29.30

Two long straight thin wires with current lie against an equally long plastic cylinder, at radius $R = 20.0$cm from the cylinders central axis. Figure 29-57a shows, in cross section, the cylinder and wire 1 but not wire 2. With wire 2 fixed in place, wire 1 is moved around the cylinder, from angle $\theta_1 = 0°$ to angle $\theta_1 = 180°$, through the first and second quadrants of the xy coordinate system. The net magnetic field $\vec{B}$ at the center of the cylinder is measured as a function of θ_1. Figure 29-57b gives the x component B_x of that field as a function of θ_1 (the vertical scale is set by $B_{xs} = 6.0\mu$T), and Fig. 29-57c gives the y component B_y (the vertical scale is set by $B_{ys} = 4.0\mu$T). (a) At what angle θ_2 is wire 2 located? What are the (b) size and (c) direction (into or out of the page) of the current

in wire 1 and the (d) size and (e) direction of the current in wire 2?

Let us call i_1 and i_2 the currents in wire 1 and 1 respectively.
Look at plot (b). We can see that

- When $\theta_1 = 0$ and $\theta_1 = 90°$, B due to wire 1 can have no x-component.
- Yet, in the above case, there is a net $B_x = 2\mu\text{T}$ which must be due to wire 2.
- When $\theta_1 = 90°$, B_x is intensified to $6\mu\text{T}$, an increase of $4\mu\text{T}$ just due to wire 1.
- Because wire 1 causes $+4\mu\text{T}$ at $\theta_1 = 90°$, i_1 **must be out of the page.**

Look at plot (c). We can see that

- When $theta_1 = 90°$, wire 1 can contribute nothing to B_y. We see that for $theta_1 = 90°$, the total $B_y = 0$ as well, so wire 2 must not contibute to B_y at all.
- Now I know that wire 2 lies on the y-axis, at either $\theta_2 = 90°$ or $270°$.
- We can choose either, but let's say wire 2 is at $\theta_2 = 270°$
- Since wire 2 contributes to B_x in the positive difection, i_2 **must be into the page.**

We know wire 2 contributes $2\mu\text{T}$ to B_x, so we use the expression for the field a distance R from a wire to find i_2:

$$B_{2,x} = 2\mu\text{T} \ = \frac{\mu_0 i_2}{2\pi R}$$

$$\frac{2\pi B_{2,x} R}{\mu_0} = i_2$$

$$\frac{2\pi\,(2\mu\text{T})\,(20\text{cm})}{4\pi 10^{-7}\,(\text{ T m / A})} = 2.0\text{A}$$

We also know wire 1 contributes $4\mu\text{T}$ to B_x, so you might intuit the current is double that of wire 2. Indeed, the same calculation as above with $B_{1,x} = 4\mu\text{T}$ instead yields $i_1 = 4.0\text{A}$.

4 29.39

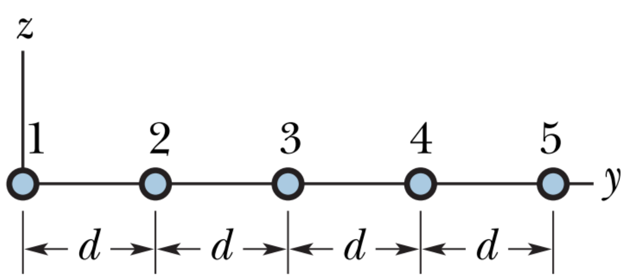

Figure 4: Problem 29.39

In Fig. 29-63, five long parallel wires in an xy plane are separated by distance $d = 50.0$cm. The currents into the page are $i_1 = 2.00$A, $i_3 = 0.250$A, $i_4 = 4.00$A, and $i_5 = 2.00$A; the current out of the page is $i_2 = 4.00$A. What is the magnitude of the net force per unit length acting on wire 3 due to the currents in the other wires?

The first thing you should do is carefully label all the information given to you on your diagram as shown in the modified diagram with red markings (this is what you would pencil in).

The circle with a dot represents a field coming out of the page. The circle with an "X" represents a field going into the page. This convention arises because it is thought that the circle with a dot in the middle looks like an arrow head approaching your face; the circle with the "X" looks like the vanes of an arrow - what you'd see of an arrow moving away from you.

Anyway, the job is to calculate the force on a wire. Our knee-jerk reaction is to recall that a current carrying wire exposed to an extermal $\vec{B}$ field is

$$\vec{F} = i\left(\vec{\ell} \times \vec{B}\right)$$

where $\vec{\ell}$ is the current direction which is situated in some external $\vec{B}$. In our case, we want to look at the force on wire 3, which has a current of 0.25A

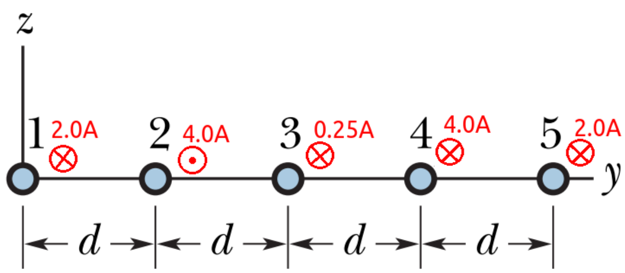

Fig. 29-63 Problems 36 and 39.

Figure 5: Problem 29.39, you should draw these currents and symbols on your paper.

into the page.

The force on i_3 will result from the total $\vec{B}$ that wire 3 experiences due to the B fields produced by all the other wires. The recipe is:

- We calculate the $\vec{B}$ field $\vec{B_1}$ due to i_1 at the position of wire 3.
- We calculate the $\vec{B}$ field $\vec{B_2}$ due to i_2 at the position of wire 3.
- We calculate the $\vec{B}$ field $\vec{B_4}$ due to i_4 at the position of wire 3.
- We calculate the $\vec{B}$ field $\vec{B_5}$ due to i_5 at the position of wire 3.
- We sum to get effective field at wire 3: $\vec{B_{eff}} = \vec{B_1} + \vec{B_2} + \vec{B_4} + \vec{B_5}$
- We use $\vec{B_{eff}}$ in the equation for the force on wire 3: $\vec{F_{on3}} = i_3 \left(\vec{\ell_3} \times \vec{B_{eff}}\right)$

Starting with $\vec{B_1}$:

The $\vec{B}$ field due to i_1 at the position of wire 3 is found by Ampere's law. We know this result: $\vec{B}$ due to a current carrying wire at a distance R from the wire has the magnitude

$$B = \frac{\mu_0 i_1}{2\pi R}$$

and a direction given by the right hand rule: point your right thumb in the direction of the current, and curl your fingers around to your palm: the field that current produces wraps around the wire in the direction of the curly fingers.

Anyway, getting the field due to wire 1 at the position of wire 3 requires we use the distance $R = 2d$

$$B_1 = \frac{\mu_0 i_1}{2\pi\,(2d)}$$

plugging in the numbers, and noting since i_1 is into the page, $\vec{B}$ will point in the $-\hat{z}$ direction (down) at wire 3:

$$\vec{B_1} = \frac{\left(4\pi 10^{-7}\text{ T m /A}\right)(2.0\text{A})}{4\pi 0.5\text{m}}\,(-\hat{z}) = 4.0\ 10^{-7}\text{ T}\,(\text{down})$$

While the field due to i_1 points down at wire 3, a quick examination with the right hand rule reveals that the field from i_1 points down at wire3, but the fields due to i_2, i_4, and i_5 all point up. So B_1 will subtract and the others will add to the effective B_{eff}.

Now we are ready to truly grind this out. The distances of each from wire 3 are

$$d_1 = 2d = 1\text{m}$$
$$d_2 = d = 0.5\text{m}$$
$$d_4 = d = 0.5\text{m}$$
$$d_5 = 2d = 1\text{m}$$

The current magnitudes, again, are

$$i_1 = 2.0\text{A}$$
$$i_2 = 4.0\text{A}$$
$$i_4 = 4.0\text{A}$$
$$i_5 = 2.0\text{A}$$

So recalling only $\vec{B_1}$ points down, the B's are

$$B_1 = -\frac{\mu_0 i_1}{2\pi d_1} = -4.0\text{e-7 T}$$

$$B_2 = +\frac{\mu_0 i_2}{2\pi d_2} = +1.6\text{e-6 T}$$

$$B_4 = +\frac{\mu_0 i_4}{2\pi d_4} = +1.6\text{e-6 T}$$

$$B_5 = +\frac{\mu_0 i_5}{2\pi d_5} = +4.0\text{e-7 T}$$

Therefore the total effective B_{eff} at the position of wire 3 is

$$B_{eff} = B_1 + B_2 + B_3 + B_4 = 3.2\text{e-6 T up}$$

and so the magnitude of the force on wire 3 is

$$F_{on3} = i_3 \left(\ell \times B_{eff}\right)$$

We don't have ℓ which is why we are asked the force per unit length:

$$\frac{F_{on3}}{\ell} = i_3 B_{eff} = 8.00 \text{ e-7 N/m}$$

5 29.41

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6 29.48

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7 29.55

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8 29.62

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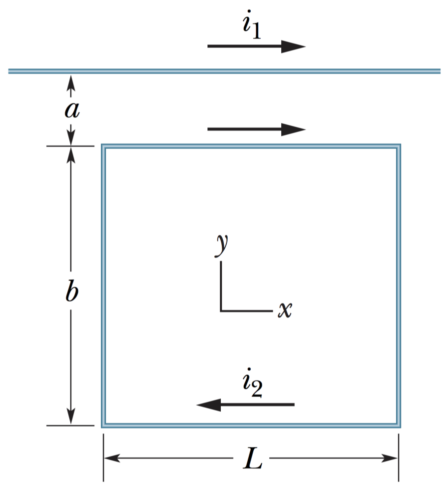

Fig. 29-65 Problem 41.

Figure 6: Problem 29.41

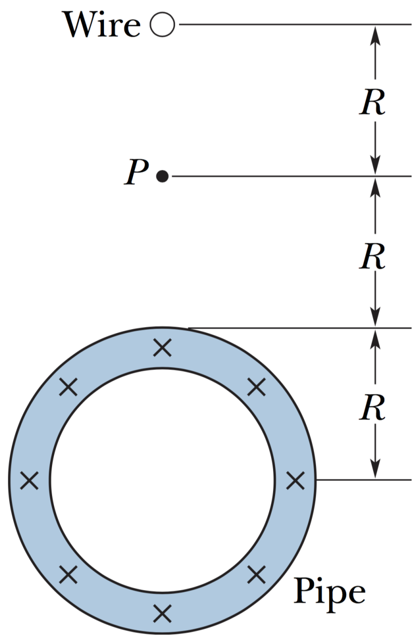

Fig. 29-70
Problem 48.

Figure 7: Problem 29.48

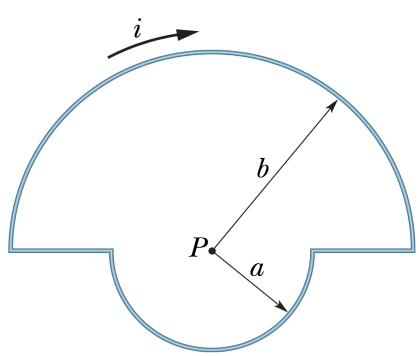

Fig. 29-74 Problem 62.

Figure 8: Problem 29.62