Design of Impedance Matching Networks for NMR and NQR Studies in the HF Band

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Abstract

For more than two decades [1], room temperature Nuclear Quadrupole Resonance (NQR) has been investiagted for land mine and general explosives detection due to its ability to unambiguously identify molecular strucutes, but poor S/N and other practical considerations have limited its use. Achieving impedance matching for a magnetic resonance probe that is sufficiently broad band and well within the dynamic range of real capacitors can be problematic for low frequency magnetic resonance studies of any kind - NQR or low field NMR. Tuning and matching is often done by hand since automation in a two parameter space is non-trivial. We present exact solutions for the completely general case of matching an inductive load $j\omega L$to some characteristic impedance Z_0, expressing the capacitance values C_1, C_2 indivdually as functions of L, r,and Z_0 and provide software for analysis.

1 Introduction

Without cryogenics or the need for large magnetic fields, room temperature table-top NMR and on home-made RF spectrometers offers high modularity

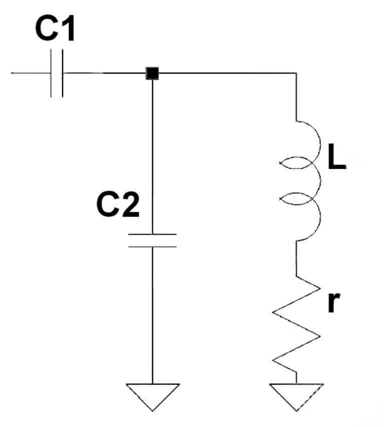

Figure 1: A typical mathching topology in NMR which can be solved exactly.

of the systems and low cost. However, the design of sufficiently sensitive probes for room temperature NMR or NQR studies in the HF band requires careful selection of components for coil tuning and impedance matching between the transmitter and the sample. For a real inductor having a finite Q, the reflection coefficient and hence the overall S/N depends on the precise adjustment of two capacitances in a non-linear fashion. For experiments not performed at fixed frequency, such as searching for an unknown NQR, the feasibility of tuning and matching can be prohibitively difficult within the dynamic range of actual capacitors, requiring frequent tear downs during which irreversible changes are introduced to the design. Most large air-variable capacitors have a dynamic range, when mounted, of about 90 percent of their max value, with the lower bound set by the parasitic capacitance between the rotor and the plates as well as the enclosure. Air gaps are available with max values from a fraction of a pF (10 turn trimmer type) to a maximum of 1nF. The ability of your air gap capacitors contraints the frequency range of operation of a given probe configuration.

We will examine the impedance of an L network to characterize the parameter space of the variables C_1, C_2, ω, L, r . The usefulness of such a calculation is to be able to predict C_1 and C_2 that result in no reflection for a target frequency and a physical coil whose impedance is presumed to be $j\omega L + r$. We express C_1 and C_2 in terms of the L, r and ω, since the NQR experimenter is not free to choose ω, and generally L and r are not independent. We will compare the results with an approximate soltuon from literature.

2 Preliminaries: the matching condition

We define the matching condition for maxiumum power transfer so tha the the imedance looking into the probe tank $Z_0 = 50\Omega$, mathching the characterizstic impedance Z_0 of the coaxial cables commonly used. To do so we must enforce the conditions

$$\begin{aligned} \text{Re}[Z] &= Z_0 \\ \text{Im}[Z] &= 0\ \Omega \\ Z_0 &= 50\ \Omega \end{aligned} \tag{1}$$

Where under typical laborator conditions, $Z_0 = 50\Omega$, but is not constrained this in analysis. The total impedance looking into the tank **??** is

$$Z = \frac{1}{\frac{1}{j\omega L + r} + j\omega C_2} - \frac{j}{\omega C_1} \tag{2}$$

Because the second term in the expression for Z has no real part, so we can treat the first term only to find Re[Z] and work backwords to express Im[Z]

$$\begin{aligned} \frac{1}{jC_2\omega + \frac{1}{r + jL\omega}} &= j\omega \frac{L(1 - C_2 L\omega^2) - C_2 r^2}{(1 - \omega^2 LC_2)^2 + (\omega C_2 r)^2} \\ &+ \frac{r}{(1 - \omega^2 LC_2)^2 + (\omega C_2 r)^2} \\ \text{Re}[Z] &= \frac{r}{(1 - \omega^2 LC_2)^2 + (\omega C_2 r)^2} \\ \text{Im}[Z] &= \omega \frac{L(1 - C_2 L\omega^2) - C_2 r^2}{(1 - \omega^2 LC_2)^2 + (\omega C_2 r)^2} - \frac{1}{\omega C_1} \end{aligned} \tag{3}$$

and so the impedance can be written in the more illumiating form

$$Z = \frac{r}{(1-\omega^2 LC_2)^2 + (\omega C_2 r)^2} + j\omega \frac{L(1-C_2 L\omega^2) - C_2 r^2}{(1-\omega^2 LC_2)^2 + (\omega C_2 r)^2} - \frac{j}{\omega C_1} \tag{4}$$

3 Approximate solution

An approximate soltuon for C_1, C_2 in terms can be obtained [2] by asserting that $(\omega C_2 r)^2$ and $C_2 r^2$ are small, the author neglects these terms reducing the full expansion of the total impedance Z to

$$Z \approx \frac{r}{(1-\omega^2 LC_2)^2} + j\omega \frac{L}{(1-\omega^2 LC_2)} + \frac{1}{j\omega C_1} \tag{5}$$

Setting $Z = Z_0$ and recognizing that Z_0 is real, we can express C_2 at the matching condition immediately

$$C_2 \approx \frac{1-\sqrt{r/Z_0}}{\omega^2 L} \tag{6}$$

and similliarly, C_1 is obtained with $\mathrm{Im}[Z] = 0$

$$C_1 \approx \frac{\sqrt{r/Z_0}}{\omega^2 L} \tag{7}$$

it follows that at the matching condition, r can be expressed in terrms of Z_0, C_1, C_2 alone

$$r = \frac{Z_0}{(1+\frac{C_2}{C_1})^2} \tag{8}$$

and is worth nothing that if r is set to exactly $Z_0/4$, then $C_1 = C_2$ when the circuit is tuned and matched, regardless of L, ω, or Z_0.

4 An exact solution

Looking back to equation 3, $\mathrm{Re}[Z]$ is quadratic in C_2 and has no dependence on C_1, so we can solve for C_2 outright. With $\mathrm{Re}[Z] = Z_0$ we find

$$\frac{r}{Z_0} = C_2^2 \left(L^2\omega^4 + r^2\omega^2\right) - 2C_2 L\omega^2 + 1 \tag{9}$$

We can now express C_2 in terms of L, ω, and r and Z_0 only by solving the quadratic

$$C_2^{\mp} = \frac{L\omega^2 Z_0 \mp \sqrt{L^2 r \omega^4 Z_0 - r^2 \omega^2 Z_0^2 + r^3 \omega^2 Z_0}}{L^2 \omega^4 Z_0 + r^2 \omega^2 Z_0} \tag{10}$$

Returning to equation 3, we examine $\mathrm{Im}[Z]$

$$\mathrm{Im}[Z] = \omega \frac{L(1 - C_2 L \omega^2) - C_2 r^2}{(1 - \omega^2 L C_2)^2 + (\omega C_2 r)^2} - \frac{1}{\omega C_1} \tag{11}$$

which we set to zero and immediately solve for C_1 in terms of C_2

$$C_1 = \frac{1}{\omega^2} \frac{(1 - \omega^2 L C_2)^2 + (\omega C_2 r)^2}{C_2 r^2 + C_2 L^2 \omega^2 - L} \tag{12}$$

Inserting each of the two roots of equation 9 into 12 gives

$$C_1^{\mp} = \pm \frac{r}{\sqrt{r \omega^2 Z_0 \left(L^2 \omega^2 + r^2 - r Z_0\right)}} \tag{13}$$

The negative value of C_1 is nonsense, confirming that C_2^- is the valid solution from the quadratic in C_2 - thus, C_2^+ should be thrown out leaving us with exact expressions for C_1, C_2 in terms of Z_0, L, ω, r:

$$\begin{aligned} C_2 &= \frac{L\omega^2 Z_0 - \sqrt{L^2 r \omega^4 Z_0 - r^2 \omega^2 Z_0^2 + r^3 \omega^2 Z_0}}{L^2 \omega^4 Z_0 + r^2 \omega^2 Z_0} \\ C_1 &= \frac{1}{\omega^2} \frac{(1 - \omega^2 L C_2)^2 + (\omega C_2 r)^2}{C_2 r^2 + C_2 L^2 \omega^2 - L} \\ &= \frac{r}{\sqrt{r \omega^2 Z_0 \left(L^2 \omega^2 + r^2 - r Z_0\right)}} \end{aligned} \tag{14}$$

Though calculation was motivated by applications in the HF-VHF band, the results assume nothing about parameters' magnitudes. Z_0 is not limited to 50Ω, and the capacitances are not assumed to be small. No approximations were made limiting the frequency of operation.

5 Comparison of approximate and exact expressions

The approximate solution obtained showed at the matching condition

$$C_1 \approx \frac{\sqrt{r/Z_0}}{\omega^2 L}$$
$$C_2 \approx \frac{1-\sqrt{r/Z_0}}{\omega^2 L} \tag{15}$$

By reannanging the to exact expression for C_2 we can compare the results

$$C_2 = \frac{L\omega^2 Z_0 - \sqrt{L^2 r\omega^4 Z_0 - r^2\omega^2 Z_0^2 + r^3\omega^2 Z_0}}{L^2\omega^4 Z_0 + r^2\omega^2 Z_0}$$
$$= \frac{1-\sqrt{r/Z_0 - r^2/(\omega^2 L^2) + r^3/(\omega^2 Z_0 L^2)}}{\omega^2 L + r^2/L} \tag{16}$$

and simillarly for C_1

$$C_1 = \frac{r}{\sqrt{r\omega^2 Z_0\left(L^2\omega^2 + r^2 - rZ_0\right)}}$$
$$= \frac{\sqrt{r/Z_0}}{\omega^2 L}\frac{1}{\sqrt{1 - r(Z_0 - r)/(L^2\omega^2)}} \tag{17}$$

We see that the exact value of C_1 at the matching condition can be seen as the approximate result from Jiang, multiplied by a correction factor $\frac{1}{\sqrt{1-r(Z_0-r)/(L^2\omega^2)}}$. The situation for C_2 is somewhat more complicated. C_2 will be very close to the approximate result when L is relatively large, Z_0 dominates r, or both.

6 Results

The approximate expressions for C_1, C_2 are in general most accurate when $Q = L\omega/r$ is high. Conversely, there is a large margin of error for arrangements with a $Q < 50$. The most interesting cases are when the approximate solutions fail when Q is at least 50, as a Q of 50 is a typical value for NMR or NQR experiments. These situations occur on the boundary of mathematically acceptable values of L. If L is low, the error in C_1 can exceed ten percent even when $Q > 50$. For example, $f = 1.7$ MHz, $L = .5\ \mu$ H, $r = .1\ \Omega$ gives $Q = 52$ and an error in the value of C_1 of .103 (exceeding ten percent).

f (MHz)	L (μH)	r (Ω)	Q	C_1 error	C_2 error
1.7	.500	.102	52.35	.103	.00406
2.7	.330	.110	50.89	.101	.00419
3.3	.270	.111	50.44	.102	.00418
4.3	.200	.104	51.96	.103	.00408
5.3	.129	.0827	51.92	.135	.00467
29.92	.03	.111	50.81	.100	.00412
200.3	.002	.0239	105.3	.110	.00212

7 Summary

We provide exact expressions for the values of capacitance that satisfy the matching condition for a typical L-network used in NMR and NQR probes for HF and VHF frequencies. We showed that the previously known approximate solutions can fail to accurately compute capacitances at the matching condition when L is sufficiently low, even in cases where Q exceeds 50.

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References

[1] A. N. Garroway, M. L. Buess, J. P. Yesinowski, J. B. Miller, and R. A. Krauss. Explosives detection by nuclear quadrupole resonance (nqr). In *SPIE's 1994 International Symposium on Optics, Imaging, and Instrumentation*, pages 139–149. International Society for Optics and Photonics, 1994.

[2] Y. J. Jiang. A simple method for measuring the q value of an nmr sample coil. *Journal of Magnetic Resonance*, 142(2):386–388, 2000.