Translation is a vector $\vec{u} = (u_x, u_y, u_z)$. U_θ transforms its as any other vector. The corresponding character is

$$1 + 2\cos\theta$$

Rotation by an infinitesimally small angle $\delta\phi$ is described by vector $\delta\vec{\phi}$ of magnitude $\delta\phi$ and along the axis of rotation. $\delta\vec{\phi}$ is a polar vector but, under rotations, it transforms as a polar vector. Its transformation adds another $1 + 2\cos\theta$ term to $\chi(U_\theta)$.

Subtracting $2(1 + 2\cos\phi)$ from $\chi(U_\theta)$, we obtain a character of *purely vibrational* degrees of freedom

$$\chi_v = N_a(1 + 2\cos\theta) - 2(1 + 2\cos\theta) = (N_a - 2)(1 + 2\cos\theta)$$