$D_2 = (C_{2v})$ group (rhombohedral and orthorhombic lattices)

We already know that $\hat{U}_{180}$ about z axis imposes no constraints.

180 rotation about the x axis: $x \to x$, $y \to -y$

$$\hat{U}^x_{180} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{9}$$

$$\begin{aligned}
\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} &= \hat{U}^x_{180}\hat{\sigma}\hat{U}^x_{180} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{yx} & -\sigma_{yy} \end{pmatrix} \\
&= \begin{pmatrix} \sigma_{xx} & -\sigma_{xy} \\ -\sigma_{yx} & \sigma_{yy} \end{pmatrix} \Rightarrow \sigma_{xy} = 0, \quad \sigma_{yx} = 0
\end{aligned} \tag{10}$$

$$\hat{\sigma}_{\text{rhombo/ortho}} = \begin{pmatrix} \sigma_{xx} & 0 \\ 0 & \sigma_{yy} \end{pmatrix} \tag{11}$$