

Boltzmann theory of electron transport

①

In equilibrium, the distribution of electrons is described by the Fermi function

$$f_0 = \frac{1}{e^{\frac{E_{\vec{k}}}{k_B T}} + 1}$$

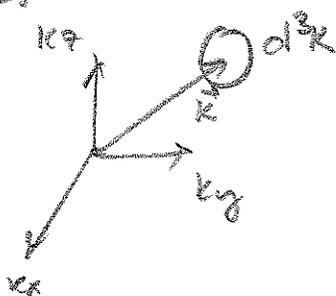
where $E_{\vec{k}}$ is the energy of an electron with wave number \vec{k} .

When external electric and magnetic fields are applied, the distribution function is not the Fermi function. If the fields vary in space and in time, the distribution function depends on wave number (\vec{k}), position (\vec{r}), and time (t).

$$f_0 \rightarrow f(\vec{k}, \vec{r}, t)$$

The meaning of function f is that

$V \cdot \frac{d^D k}{(2\pi)^D} f(\vec{k}, \vec{r}, t)$ gives a number of electrons with wavevectors located in volume $d^D k$ near point \vec{k} , as measured at point \vec{r} and at time t .



The distribution function changes from point to point in the \vec{k} -space, and is also different in different points of \vec{r} and at different instances of time.

If the distribution function is known, (2)
various macroscopic observables can be found as its moments. For example, the number density of electrons $n(\vec{r}_1, t)$ is

$$n(\vec{r}_1, t) = 2 \int \frac{d^D k}{(2\pi)^D} f(\vec{k}, \vec{r}_1, t).$$

The electric current is obtained by multiplying f by the current carried by one electron $-e\vec{v}_k$ and integrating over the momentum space.

$$\vec{j}(\vec{r}_1, t) = -2e \int \frac{d^D k}{(2\pi)^D} \vec{v}_k f(\vec{k}, \vec{r}_1, t)$$

(Here, and in the formula for n , a factor of 2 is due to spin summation.)

Notice that the equilibrium function f_0 is a function of energy only, while the energy is an even function of \vec{k} : $E_{\vec{k}} = E_{-\vec{k}}$.

On the other hand, \vec{v}_k is odd: $\vec{v}_{\vec{k}} = -\vec{v}_{-\vec{k}}$.

Therefore, the equilibrium distribution carries no net current

$$\vec{j}_0 = -2 \int \frac{d^D k}{(2\pi)^D} \vec{v}_k f_0(\vec{k}) = 0.$$

because the elementary currents carried by electrons moving in opposite directions cancel out. In equilibrium, there is an equal number of electrons with \vec{k} and $-\vec{k}$. Only non-equilibrium distributions lead to finite current.

Boltzmann proposed that the distribution function changes due to various scattering events experienced by electrons. In short, his equation reads ③

$$\frac{df}{dt} = I_c \{f\}$$

where $I_c \{f\}$, called a "collision integral" describes the effect of scattering. The total time derivative on L.H.S. is

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{d\vec{r}}{dt} \cdot \frac{\partial f}{\partial \vec{r}} + \frac{d\vec{k}}{dt} \cdot \frac{\partial f}{\partial \vec{k}}$$

where $\frac{d\vec{r}}{dt} = \vec{v}_k$

and $\frac{d\vec{k}}{dt} = \frac{1}{\hbar} \vec{F}$,

where \vec{F} is the force, e.g. electric or magnetic, acting on electrons. So, here is the Boltzmann eq-ⁿ

$$\frac{\partial f}{\partial t} + \vec{v}_k \cdot \frac{\partial f}{\partial \vec{r}} + \frac{1}{\hbar} \vec{F} \cdot \frac{\partial f}{\partial \vec{k}} = I_c \{f\}$$

Particular form of the collision integral depends on the scattering process. There is, however, a simple model, called "relaxation time approximation", in which

$$I_c \{f\} = - \frac{f(\vec{k}, t, \vec{r}) - f_0}{\tau}$$

$f_0 \leftarrow$ equilibrium function

The meaning of that approximation is very simple. External forces disturb the equilibrium distribution but collisions try to restore its original form. For example, the electric field gives electrons momentum along the field but collisions of electrons with impurities tend to randomize the motion along the field. The rate of change of f is proportional to the deviation of f from its equilibrium value with negative sign. τ is a phenomenological constant of units time which is the mean free time of electrons.

then,

$$\frac{\partial f}{\partial t} + \vec{v}_k \cdot \frac{\partial f}{\partial \vec{r}} + \frac{1}{\hbar} \vec{F} \cdot \frac{\partial f}{\partial \vec{k}} = - \frac{f - f_0}{\tau}$$

Further simplifications:

- 1) Steady-state: if the external fields are time-independent, so is $f \Rightarrow \frac{\partial f}{\partial t} = 0$
- 2) if external fields are independent of coordinates, so is $f \Rightarrow \frac{\partial f}{\partial \vec{r}} = 0$.

Then

$$\left| \frac{1}{\hbar} \vec{F} \cdot \frac{\partial f}{\partial \vec{k}} = - \frac{f - f_0}{\tau} \right|$$

Electric field only

(5)

$$\vec{F} = -e\vec{E}$$

We are interested only in weak electric fields, when the distribution function is disturbed only slightly. In that case, $\frac{\partial f}{\partial \vec{k}}$ can be replaced by

$$\frac{\partial f_0}{\partial \vec{k}} = \frac{dE}{d\vec{k}} \frac{\partial f_0}{\partial E} = \frac{\vec{v}_k}{v} \frac{\partial f_0}{\partial E} \quad \text{and we have}$$

$$-e(\vec{v}_k \cdot \vec{E}) \frac{\partial f_0}{\partial E} = -\frac{f-f_0}{\tau} \Rightarrow$$

$$f-f_0 = e\tau(\vec{v}_k \cdot \vec{E}) \frac{\partial f_0}{\partial E_k}$$

The electric current

$$\begin{aligned} \vec{j} &= -2e \int \frac{d^D k}{(2\pi)^D} \vec{v}_k f = -2e \int \frac{d^D k}{(2\pi)^D} \vec{v}_k (f-f_0) = \\ &= -2e^2 \tau \int \frac{d^D k}{(2\pi)^D} \vec{v}_k (\vec{v}_k \cdot \vec{E}) \frac{\partial f_0}{\partial E_k} \end{aligned}$$

Suppose that $\vec{E} \parallel \hat{x}$,

Then

$$j_x = -2e^2 \tau \int \frac{d^D k}{(2\pi)^D} v_x^2 \frac{\partial f_0}{\partial E} E$$

and

$$j_y = -2e^2 \tau \int \frac{d^D k}{(2\pi)^D} v_x v_y \frac{\partial f_0}{\partial E} E$$

...

add 0
to the current
↓

We see that the components of the conductivity tensor are given by

(6)

$$\sigma_{\alpha\beta} = -2e^2 T \int \frac{d^D k}{(2\pi)^D} v_{\alpha} v_{\beta} \frac{\partial f_0}{\partial E}$$

Recall that for $kT \ll E_F$, the Fermi function is almost a step function, which means that

- its derivative is almost a δ -function



Therefore, the integral in the formula for the conductivity involve only states near the Fermi surface, however complicated that surface may be!

$$\sigma_{\alpha\beta} = 2e^2 T \int \frac{d^D k}{(2\pi)^D} v_{\alpha} v_{\beta} \delta(E_k - E_F)$$

This result can be cast in even simpler form. Instead of integrating over a 3D momentum space, consider a set of isoenergetic surfaces.

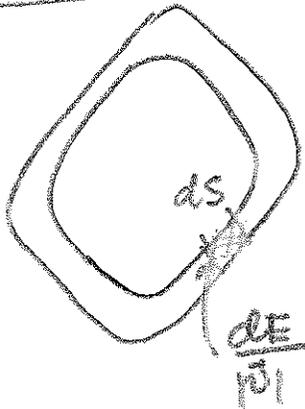
Suppose that the area element of one such surface at energy E is dS . Consider a small variation of energy $E \rightarrow E + dE$. The "distance" in the momentum space between these two surfaces

is $\frac{dE}{\hbar|\mathbf{v}|}$. The elementary volume enclosed (7)

in between these two surfaces is $\frac{dS dE}{\hbar|\mathbf{v}|}$,

Therefore,

$$\int d^3k = \int dE \frac{\int dS}{\hbar|\mathbf{v}|}$$



Two isoenergetic surfaces at energies E and $E+dE$.

Now,

$$\begin{aligned} \sigma_{\alpha\beta} &= \frac{2e^2\tau}{(2\pi)^D} \int dE \frac{\int dS}{\hbar|\mathbf{v}|} v_{\alpha} v_{\beta} \delta(E - E_F) \\ &= \frac{2e^2\tau}{(2\pi)^{D+1}} \int_{F.S.} \frac{dS}{|\mathbf{v}|} v_{\alpha} v_{\beta}, \end{aligned}$$

where F.S. below the integral mean that the integral goes over the Fermi surface.

Example: Tight-binding spectrum in 2D

Consider a 2D square lattice with spacing a .

The spectrum

$$E = -2\gamma \cos(k_x a) - 2\gamma \cos(k_y a)$$

To find, e.g., σ_{xx} , we need to know (8)

$$v_x \text{ and } |v| = \sqrt{v_x^2 + v_y^2}$$

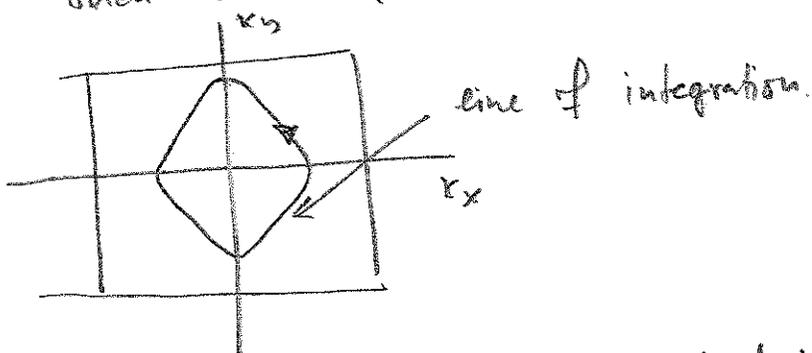
$$v_x = \frac{2\gamma a}{\hbar} \sin(k_x a)$$

$$v_y = \frac{2\gamma a}{\hbar} \sin(k_y a)$$

$$|v| = \frac{2\gamma a}{\hbar} \sqrt{\sin^2(k_x a) + \sin^2(k_y a)}$$

$$\sigma_{xx} = \frac{2e^2 \tau}{(2\pi)^2 \hbar} \int_{\text{F.S.}} ds \frac{v_x^2}{|v|} = \frac{2e^2 \tau}{(2\pi)^2 \hbar} \int_{\text{F.S.}} ds \frac{2\gamma a \frac{\sin^2 k_x a}{\sqrt{\sin^2 k_x a + \sin^2 k_y a}}}{\hbar}$$

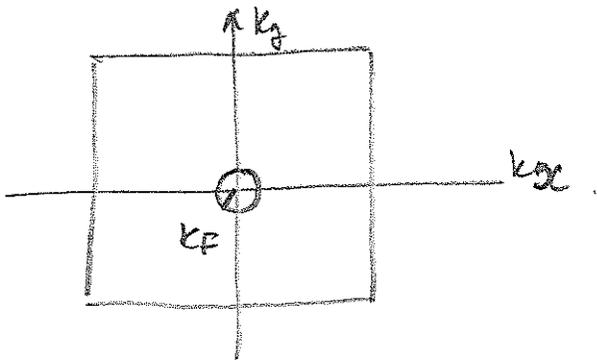
The integral goes over the Fermi surface (line) at fixed value of the Fermi energy



As two particular simple but illustrative examples, let's consider the cases of small and almost full filling.

(A) Almost empty band : $E_F \ll 2\epsilon$

In that case, the Fermi surface is a small circle at the origin



(9)

The radius of the circle is determined in the same way as for free electrons

$$2 \cdot \frac{\pi k_F^2}{(2\pi)^2} = n$$

$$k_F = \sqrt{2\pi n}$$

The "surface" (line) element

$$dS = k_F d\varphi$$

$$v_x = \frac{\hbar k_x}{m^*} = \frac{\hbar k_F}{m^*} \cos \varphi \quad ; \quad \frac{1}{m^*} = \frac{2\tau a^2}{\hbar^2}$$

$$v_y = \frac{\hbar k_y}{m^*} = \frac{\hbar k_F}{m^*} \sin \varphi$$

$$|v| = \frac{\hbar k_F}{m^*}$$

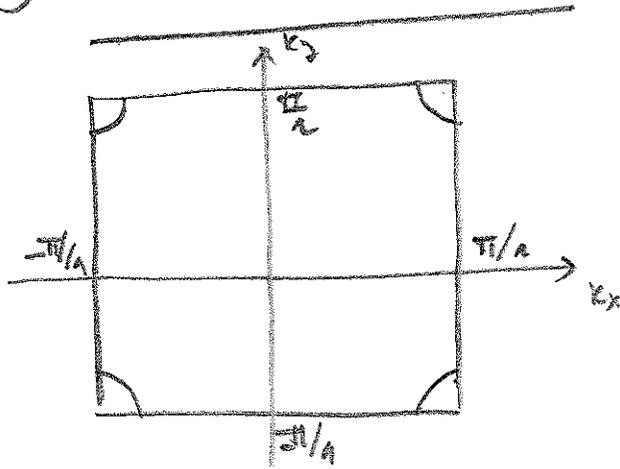
$$\sigma_{xx} = \frac{2e^2 \tau}{(2\pi)^2} \frac{1}{\hbar} \int_0^{2\pi} d\varphi k_F \frac{\hbar^2 \left(\frac{k_F}{m}\right)^2 \cos^2 \varphi}{\hbar \frac{k_F}{m}}$$

$$= \frac{e^2 \tau}{2\pi} \frac{k_F^2}{m} = \frac{e^2 \tau n}{m},$$

which is precisely the Drude formula for free electrons with effective mass m^*

(B) Almost full band

(10)



The Fermi surface is obtained by adding together four segments.

The Fermi surface consists of four segments located at the corners of the Brillouin zone. For almost full filling, the four segments, when added together, form a small circular Fermi surface. Consider one of the segments, say, near $(\pi/a, \pi/a)$. Counting wave

wavenumbers from the corner

$$k_x = \frac{\pi}{a} - q_x, \quad |q_x| \ll \frac{\pi}{a}$$

$$k_y = \frac{\pi}{a} - q_y, \quad |q_y| \ll \frac{\pi}{a}$$

The area enclosed by the four segments together can be found without any geometric construction. Indeed, full filling corresponds to 2 electrons per atom, or number density $n_0 = 2/a^2$.

When $n = n_0$, the entire B.Z. is filled

$$\frac{2 \left(\frac{2\pi}{a}\right)^2}{(2\pi)^2} = \frac{2}{a^2}$$

Suppose that the number density is somewhat smaller than n_0 (11).

$$n = n_0 - \delta n.$$

The area of the remaining part of the B.Z. is the area of the Fermi surface A_F

$$2 \frac{\left[\left(\frac{2\pi}{a} \right)^2 - A_F \right]}{(2\pi)^2} = \frac{2}{a^2} - \delta n \quad \text{or}$$

$$2 \frac{A_F}{(2\pi)^2} = \delta n$$

Writing A_F as $A_F = \pi q_F^2$, we see that the relation between q_F and δn is the same as between k_F and n :

$$\boxed{\frac{q_F^2}{2\pi} = \delta n}$$

Instead of electrons in the center of the B.Z., we now have holes near the corner of the B.Z. The difference between electrons and holes is in the sign of the effective mass. For electrons,

$$\Xi = -2\alpha \left[\cos k_x a + \cos k_y a \right] \approx$$

$$\approx -2\alpha \left[1 - \frac{k_x^2 a^2}{2} + 1 - \frac{k_y^2 a^2}{2} \right] \Rightarrow$$

Electron:

$$\frac{1}{m_{xx}^*} = \frac{2\gamma a^2}{\hbar^2} = \frac{1}{m^*}$$

$$\frac{1}{m_{yy}^*} = \frac{2\gamma a^2}{\hbar^2} = \frac{1}{m^*}$$

(12)

For holes

$$\begin{aligned} E &= -2\gamma [\cos k_x a + \cos k_y a] = -2\gamma \left[\cos \left(\frac{\pi}{a} - q_x \right) a \right] + \\ &+ \cos \left[\left(\frac{\pi}{a} - q_y \right) a \right] = 2\gamma [\cos q_x a + \cos q_y a] \\ &\approx 2\gamma \left[1 - \frac{1}{2} q_x^2 a^2 + 1 - \frac{1}{2} q_y^2 a^2 \right] \Rightarrow \end{aligned}$$

holes:

$$\frac{1}{m_{xx}^*} = -\frac{2\gamma a^2}{\hbar^2} = -\frac{1}{m^*}$$

$$\frac{1}{m_{yy}^*} = -\frac{2\gamma a^2}{\hbar^2} = -\frac{1}{m^*}$$

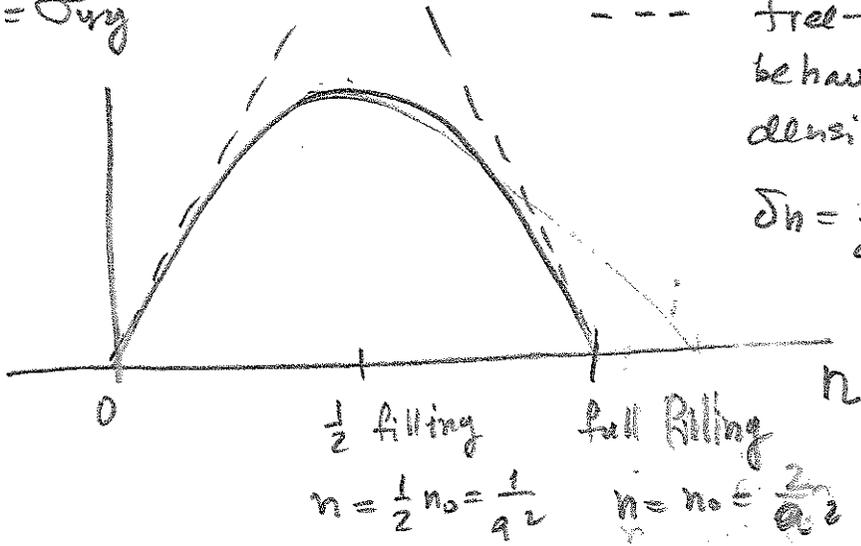
However, the effective mass enters only as an absolute value

$$\text{via } \frac{v_x^2}{|v|} = \frac{k_x^2}{(m^*)^2} \frac{1}{|k|} = \frac{k_x}{k} \frac{1}{|m^*|} = \begin{cases} \frac{k_F}{|m^*|}, \text{ electrons} \\ \frac{q_F}{|m^*|}, \text{ holes.} \end{cases}$$

Therefore, the conductivity of almost full band is equivalent to that of almost empty band upon replacement $n \rightarrow \delta n$.

$$\sigma_{xx} = \sigma_{yy} = \begin{cases} \int \frac{e^2 \cdot n \cdot \tau}{|m^*|}, & \text{almost empty band} \\ \int \frac{e^2 \cdot \delta n \cdot \tau}{|m^*|}, & \text{almost full band.} \end{cases}$$

$$\sigma_{xx} = \sigma_{yy}$$



--- free-like behavior with density n or $\delta n = \frac{z}{a^2} - n$.

In other words, the conductivity is a symmetric function of the electron number density about a $\frac{1}{2}$ -filling point.

If the band is not either almost empty or almost full, the result is significantly different from $\sigma = \frac{e^2 n v}{m v}$, because the effective mass depend on k^* (and, therefore, E_f) at arbitrary filling. It is quite easy to find σ at $\frac{1}{2}$ filling - do it as an exercise.

An important note:

(4)

In our treatment, holes occur as electron states with negative effective mass. That is a rigorous way of thinking about holes. However, it is a common practice to view holes as particles with positive mass and positive charge. This is a legitimate point of view, if one defines the acceleration as (the α component of)

$$a_{\alpha} = \frac{d}{dt} v_{\alpha} = \sum_{\beta} \frac{\partial v_{\alpha}}{\partial k_{\beta}} \frac{dk_{\beta}}{dt} = \sum_{\beta} \frac{\partial^2 E}{\hbar^2 \partial k_{\alpha} \partial k_{\beta}} \left(\frac{1}{\hbar} \right) F_{\beta}$$
$$= \sum_{\beta} \frac{1}{m_{\alpha\beta}} F_{\beta}$$

where

$$\frac{1}{m_{\alpha\beta}} = \frac{1}{\hbar^2} \frac{\partial^2 E}{\partial k_{\alpha} \partial k_{\beta}}$$

For electric and magnetic forces

$$F_{\beta} = -e E_{\alpha} + (-e) (\vec{v} \times \vec{B})_{\alpha}$$

and

$$a_{\alpha} = \sum_{\beta} \frac{(-e)}{m_{\alpha\beta}} \left(E_{\alpha} + (\vec{v} \times \vec{B})_{\alpha} \right).$$

Therefore, only the ratio $\frac{(-e)}{m_{\alpha\beta}}$ enters the acceleration, and one can flip the signs of e and $m_{\alpha\beta}$

at the same time. Then, for hole-like states (15)

$$a_{\alpha} = \sum_{\beta} \frac{-e}{-|m_{\alpha\beta}|} (\mathcal{E}_{\alpha} + (\vec{v}_{\alpha} \times \vec{B})_{\alpha}) =$$
$$= \sum_{\beta} \frac{(+e)}{|m_{\alpha\beta}|} (\mathcal{E}_{\alpha} + (\vec{v}_{\alpha} \times \vec{B})_{\alpha})$$

which does describe the motion of charges $+e$ with positive mass.

Although this is a legitimate procedure, one has to keep in mind that:

- 1) if the force is not electric, holes still move as if their masses are negative.
- 2) the acceleration per se does not enter our formalism, so this concept is, albeit useful, extraneous.
- 3) at any rate, one should not change the sign twice! That is, thinking of holes as particles with both positive charge and negative mass is entirely wrong!

Hall conductivity from the Boltzmann eq-n (16)

Now we can restore the magnetic component of the force. Our main goal is to see how states with negative m^* manifest themselves in a Hall effect measurement. Recall that the free-electron model gives for the Hall constant

$$R_H = -\frac{1}{en},$$

which is always negative. How can the sign be changed?

Go back to the Boltzmann eq-n

$$-e(\vec{v} \cdot \vec{E}) \frac{\partial f}{\partial E} - e \frac{(\vec{v} \times \vec{B})}{\hbar} \cdot \frac{\partial f}{\partial \vec{k}} = -\frac{f - f_0}{\tau}$$

If I now try to replace f in $\frac{\partial f}{\partial \vec{k}}$ by the equilibrium distribution function f_0 , the result would be that the magnetic field drops out. Indeed,

$$(\vec{v} \times \vec{B}) \cdot \frac{\partial f_0}{\partial \vec{k}} = (\vec{v} \times \vec{B}) \cdot \vec{v} \frac{\partial f_0}{\partial E} = 0$$

because $\vec{v} \times \vec{B} \perp \vec{v}$.

Hence, we need to keep non-equilibrium part of f in the magnetic-field part.

Denote

$$g \equiv f - f_0$$

$$-e (\vec{v} \cdot \vec{E}) \frac{\partial f_0}{\partial E} - e \frac{(\vec{v} \times \vec{B})}{\hbar} \cdot \frac{\partial g}{\partial \vec{k}} = - \frac{g}{\tau}$$

I am going to treat \vec{B} as a small perturbation,

Accordingly,

$$g = g^{(0)} + g^{(1)} + \dots,$$

where $g^{(0)} \sim U(B^0)$ and $g^{(1)} \sim U(B)$.

$$-e (\vec{v} \cdot \vec{E}) \frac{\partial f_0}{\partial E} - e \frac{(\vec{v} \times \vec{B})}{\hbar} \cdot \frac{\partial}{\partial \vec{k}} (g^{(0)} + g^{(1)}) = - \frac{g^{(0)} + g^{(1)}}{\tau}$$

At the first iteration, neglect the ⁽⁰⁾entire magnetic-field term on the left and $g^{(1)}$ on the right:

$$-e (\vec{v} \cdot \vec{E}) \frac{\partial f_0}{\partial E} = - \frac{g^{(0)}}{\tau} \rightarrow (1^{st} \text{ iteration}).$$

$$g^{(0)} = e\tau (\vec{v} \cdot \vec{E}) \frac{\partial f_0}{\partial E}$$

That is the same correction to the distribution function we found in the absence of \vec{B} . Now, substitute $g^{(0)}$ in the magnetic-field term but still neglect $g^{(1)}$ there - keeping the product $B g^{(0)}$ would be of second order in B^2 .

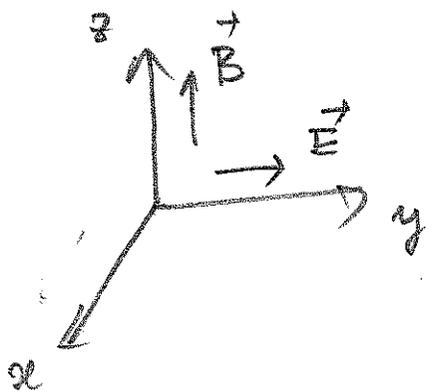
$$-e \frac{(\vec{v} \times \vec{B})}{\hbar} \cdot \frac{\partial g^{(0)}}{\partial \vec{k}} = - \frac{g^{(1)}}{\tau}$$

$$g^{(1)} = e\tau \frac{(\vec{v} \times \vec{B})}{\hbar} \cdot \frac{\partial}{\partial \vec{k}} g^{(0)} = e\tau \frac{(\vec{v} \times \vec{B})}{\hbar} \cdot \frac{\partial}{\partial \vec{k}} \left(e\tau (\vec{v} \cdot \vec{E}) \frac{\partial f_0}{\partial E} \right)$$

when $\frac{\partial}{\partial \vec{k}}$ acts on $\frac{\partial P_0}{\partial E}$, it produces a term proportional to \vec{v} , which kills the Lorentz force. Therefore, $\frac{\partial}{\partial \vec{k}}$ needs to act only on $\vec{v} \cdot \vec{E}$.

$$g''' = e^2 \tau^2 \frac{\partial P_0}{\partial E} \frac{(\vec{v} \times \vec{B})}{\hbar} \cdot \frac{\partial}{\partial \vec{k}} (\vec{v} \cdot \vec{E})$$

Consider the Hall-effect geometry



Then,

$$\vec{v} \times \vec{B} = v_y B \hat{x} - v_x B \hat{y}$$

$$\vec{v} \cdot \vec{E} = v_y E$$

$$\frac{\partial}{\partial \vec{k}} (\vec{v} \cdot \vec{E}) = E \frac{\partial}{\partial \vec{k}} v_y = E \left[\frac{\partial v_y}{\partial k_x} \hat{x} + \frac{\partial v_y}{\partial k_y} \hat{y} \right]$$

and

$$g''' = \frac{e^2 \tau^2}{\hbar} \frac{\partial P_0}{\partial E} E B \left(v_y \hat{x} - v_x \hat{y} \right) \left(\frac{\partial v_y}{\partial k_x} \hat{x} + \frac{\partial v_y}{\partial k_y} \hat{y} \right)$$

$$= \frac{e^2 \tau^2}{\hbar} \frac{\partial P_0}{\partial E} E B \left(v_y \frac{\partial v_y}{\partial k_x} - v_x \frac{\partial v_y}{\partial k_y} \right)$$

Notice that the effective masses occurred:

$$\frac{\partial v_y}{\partial k_x} = \hbar \frac{1}{m_{yx}}$$

$$\frac{\partial v_x}{\partial k_y} = \hbar \frac{1}{m_{xy}}$$

$$g^{(1)} = e^2 \tau^2 \frac{\partial f_0}{\partial E} \epsilon B \left(v_y \frac{1}{m_{yx}} - v_x \frac{1}{m_{xy}} \right).$$

The Hall current

$$j_x = -2e \int \frac{d^D k}{(2\pi)^D} v_x g^{(1)}$$

is proportional to ϵ , therefore, the Hall conductivity

$$\sigma_{xy} = j_x / \epsilon \text{ can be read off as}$$

$$\sigma_{xy} = -2 \left(e^3 \tau^2 B \int \frac{d^D k}{(2\pi)^D} v_x \left(v_y \frac{1}{m_{yx}} - v_x \frac{1}{m_{xy}} \right) \frac{\partial f_0}{\partial E} \right)$$

Notice that $e > 0$ but the effective masses can be of either sign. In the tight-binding model, the off-diagonal components $\frac{1}{m_{\alpha \neq \beta}} = 0$. Then,

$$\sigma_{xy} = 2 \left(e^3 \tau^2 B \int \frac{d^D k}{(2\pi)^D} v_x^2 \frac{1}{m_{yy}} \frac{\partial f_0}{\partial E} \right)$$

The integral over the momentum space can be cast into the integral over the Fermi surface in the same way it was done in the absence of the field.

$$\begin{aligned}\sigma_{xy} &= 2 \frac{e^3}{\hbar} q^2 B \cdot \int \frac{dS}{4\pi} v_x^2 \frac{1}{m_{yy}} \\ &= \frac{2e^3 \tau^2 B}{\hbar} \cdot \int \frac{dS}{4\pi} \frac{v_x^2}{m_{yy}}.\end{aligned}$$

Now, the reason of the change in sign is obvious.

For almost empty band, $m_{yy} = m^*$

and

$$\sigma_{xy} = \frac{eB\tau}{m^*} \cdot \frac{2e^2\tau}{\hbar} \int \frac{dS}{4\pi} v_x^2 = \frac{eB\tau}{m^*} \cdot \sigma_{xx} = \omega_{cT} \sigma_{xx}$$

where σ_{xx} is the conductivity in the absence of B .

For almost full band, $m_{yy} = -1m^*$

and

$$\underline{\sigma_{xy} = -\omega_{cT} \sigma_{xx}}$$

Therefore, whether R_H is positive or negative depends on the filling! Less than $\frac{1}{2}$ full bands give electron-like sign of R_H , more than $\frac{1}{2}$ full bands give hole-like sign of R_H .

As a corollary, $R_H = 0$ in a $\frac{1}{2}$ -full band

this may seem quite paradoxical because at $\frac{1}{2}$ -filling, the system is a perfect metal. But it means that the electron- and hole contributions to the Hall current cancel each other out.