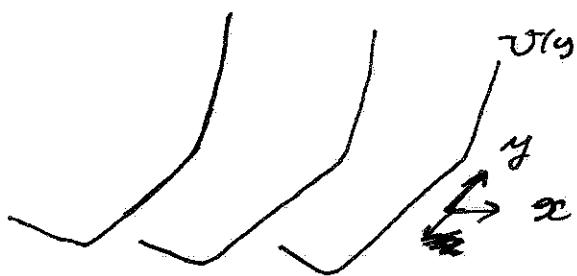


Edge state picture of Integer Quantum Hall Effect. ①

Edge states occur in a finite sample. Near the boundaries, degeneracy of Landau levels with respect to the center-of-oscillator position is lifted, and the states acquire dispersion in k_x .



$V(y)$: confining potential in the y -direction

Suppose that $V(y)$ varies on a scale much larger than the

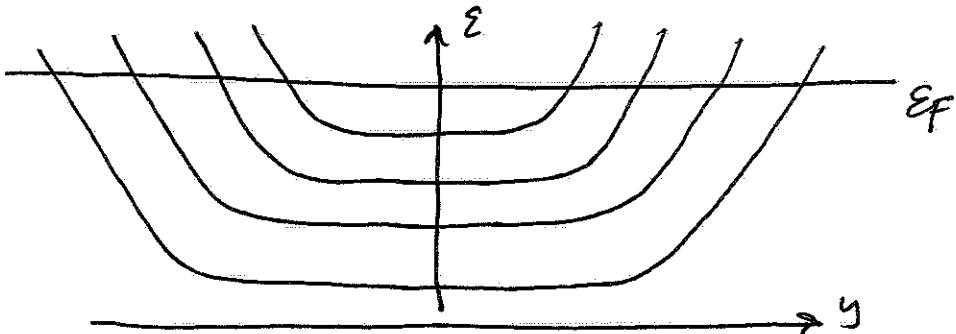
magnetic length. Then the effect of the boundary can be taken into account semiclassically. The y -component of the wavefunction now satisfies the Schrödinger eqn

$$(H_{SO}(y) + V(y))\psi(y) = E \psi(y)$$

where H_{SO} is the Hamiltonian of a simple harmonic oscillator which leads to Landau quantization. For slowly varying V , the energy of the state can be written as

$$E_{nk_x} = \hbar \omega_c (n + \frac{1}{2}) + V(y_0),$$

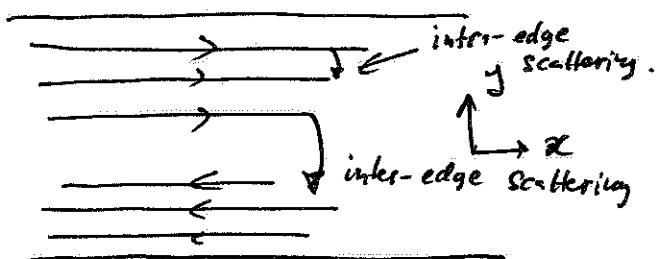
where $y_0 = k_x \hbar^2$ is the center-of-oscillator position.



Landau levels in a finite sample

Because E_{kx} now depends on k_x , the group velocity of the state is non-zero. ②

$$v_x = \frac{1}{\hbar} \frac{\partial E_{kx}}{\partial k_x} = \frac{1}{\hbar} \frac{\partial}{\partial k_x} V(k_x l_B^2) = \frac{1}{\hbar} l_B^2 V'(k_x l_B^2)$$



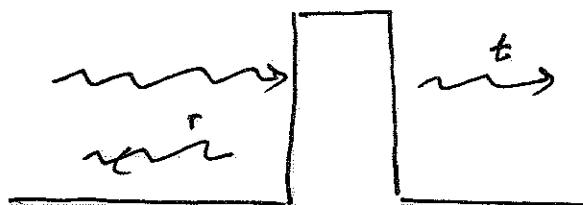
The states are localized at the points where E_F crosses the Landau level.

Edge states; top view. This is nothing more than states that carry net diamagnetic current in response to the external magnetic field.

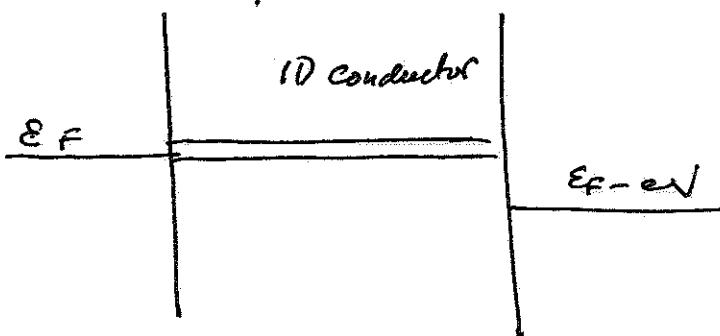
Edge states are robust with respect to disorder. Namely, intra-edge scattering does not change the total number of electrons moving along the given edge. Scattering between opposite edges requires tunneling through the bulk of the sample and is thus negligibly small in macroscopically large samples (unless E_F touches the bottom of the Landau level). Since electrons travel freely along the edges, the voltage difference between any two points on the same edge is zero, thus, $V_{xx} = 0$ at finite current, and $P_{xx} = 0$ when E_F is in between the Landau levels. This is one of the manifestations of the quantum Hall effect. In order to see why S_{xy} is quantized, we need to invoke \Rightarrow

Landauer formula

Text-book problem : potential barrier



How to measure $|t|$ and $|r|$?



Current of right-moving carriers

$$I_R = +2e \int_0^{\infty} \frac{dk}{2\pi} \nu f(E_F)$$

Spin ↑

$$I_L = -2e \int_0^{\infty} \frac{dk}{2\pi} \nu f(E_F - ev)$$

Total current

$$I = I_R + I_L = -2e \int_0^{\infty} \frac{dk}{2\pi} \nu [f(E_F) - f(E_F - ev)]$$

$$dk = dE \left| \frac{dk}{dE} \right| = \frac{1}{v} dE \frac{1}{v}$$

$$I = -\frac{2e^2}{2\pi v} \int_0^{\infty} dE [f(E_F) - f(E_F - ev)]$$

$$f(E_F) - f(E_F - ev) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} - \frac{1}{e^{\frac{E-E_F+ev}{kT} + 1}}$$

$\stackrel{\text{small } V}{=} \frac{\frac{\partial f}{\partial E} eV}{e^{\frac{E-E_F}{kT}} + 1}$
 $= \left(-\frac{\partial f}{\partial E} \right) eV$

4)

$$I = \frac{2e^2}{h} \int_0^\infty dE \left(-\frac{\partial f}{\partial E} \right) eV = \frac{2e^2}{h} eV$$

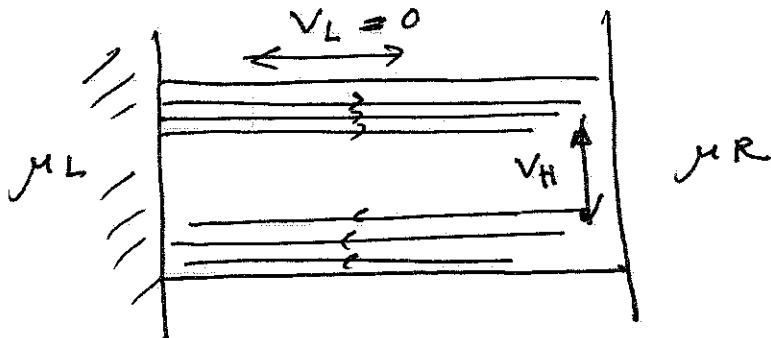
\downarrow
1 at any T

$$G = \frac{I}{V} = \frac{2e^2}{h}$$

conductance of N ballistic channels

$$G = \frac{2e^2}{h} \cdot N$$

Back to IQHE



$$\text{Hall voltage : } eV_H = \mu_L - \mu_R$$

$$\text{Total current } I = \frac{2e^2}{h} \cdot N (\mu_L - \mu_R)$$

$$\text{Hall resistance } R_H = \frac{V_H}{I} = \frac{(\mu_L - \mu_R)e}{\frac{2e^2}{h} N (\mu_L - \mu_R)} = \frac{h}{2e^2} \frac{1}{N}$$